Temporal and spectral characteristics of the LEUTL SASE FEL

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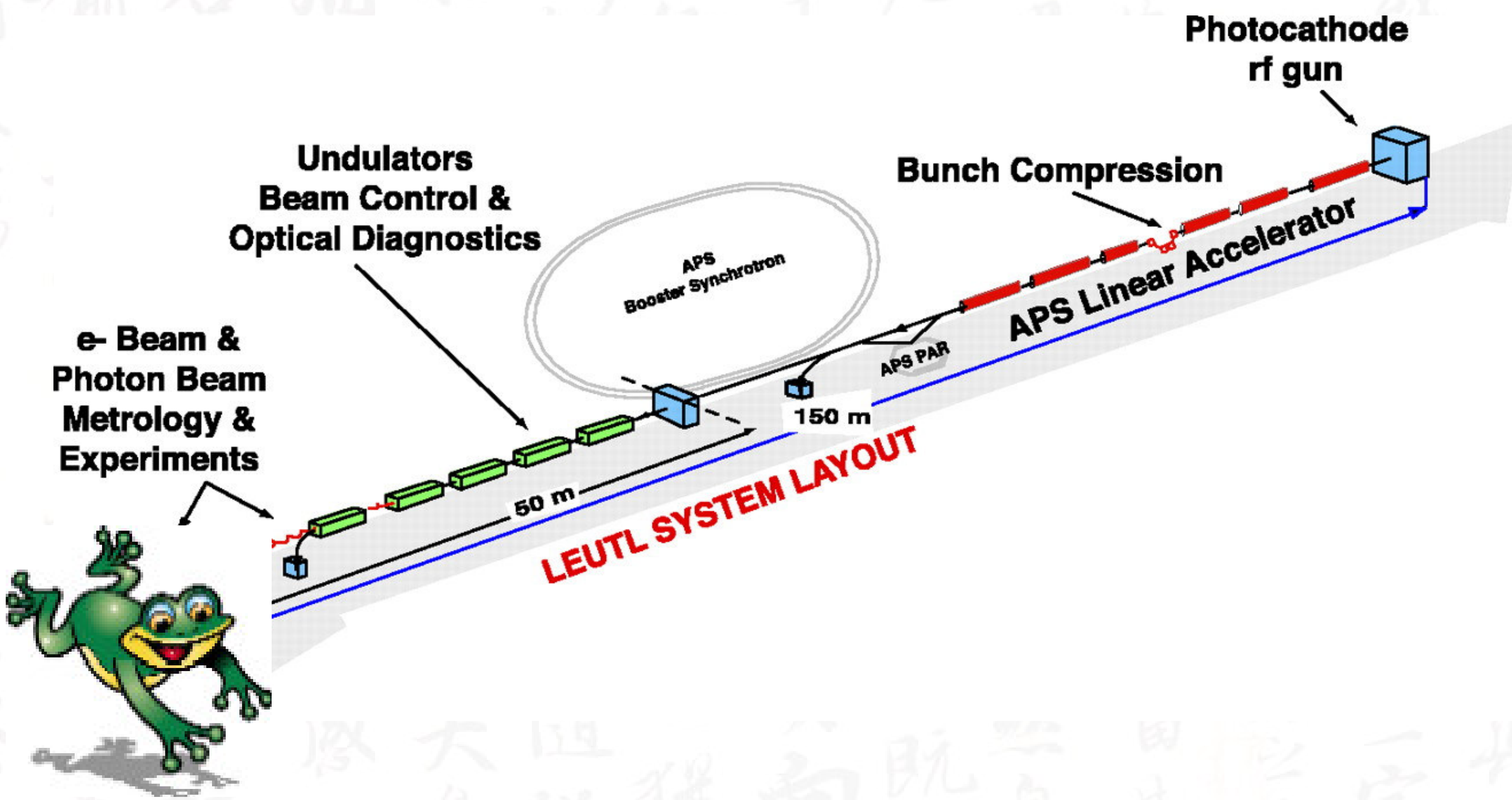
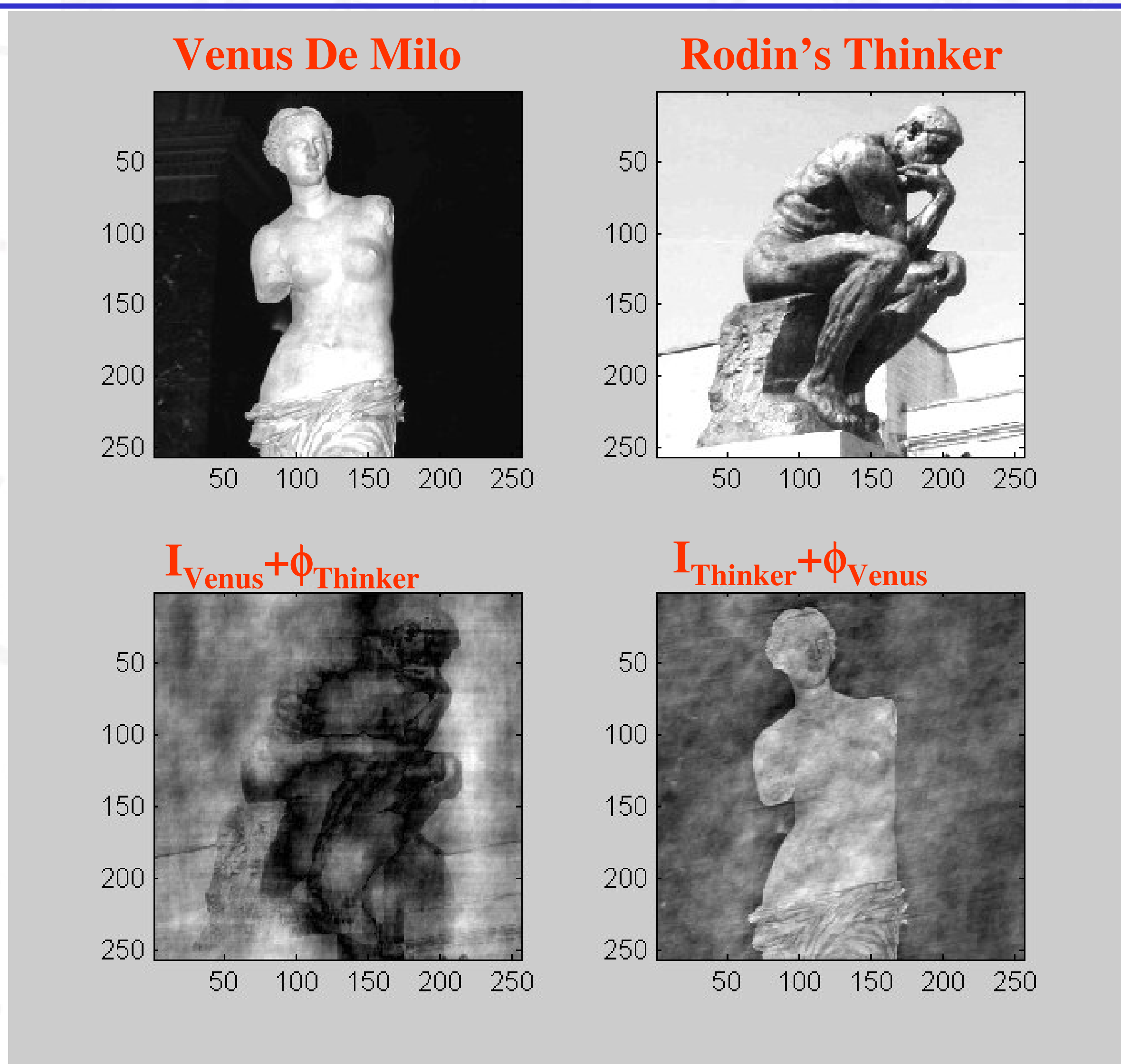
The low-energy undulator test line FEL: future ALFF

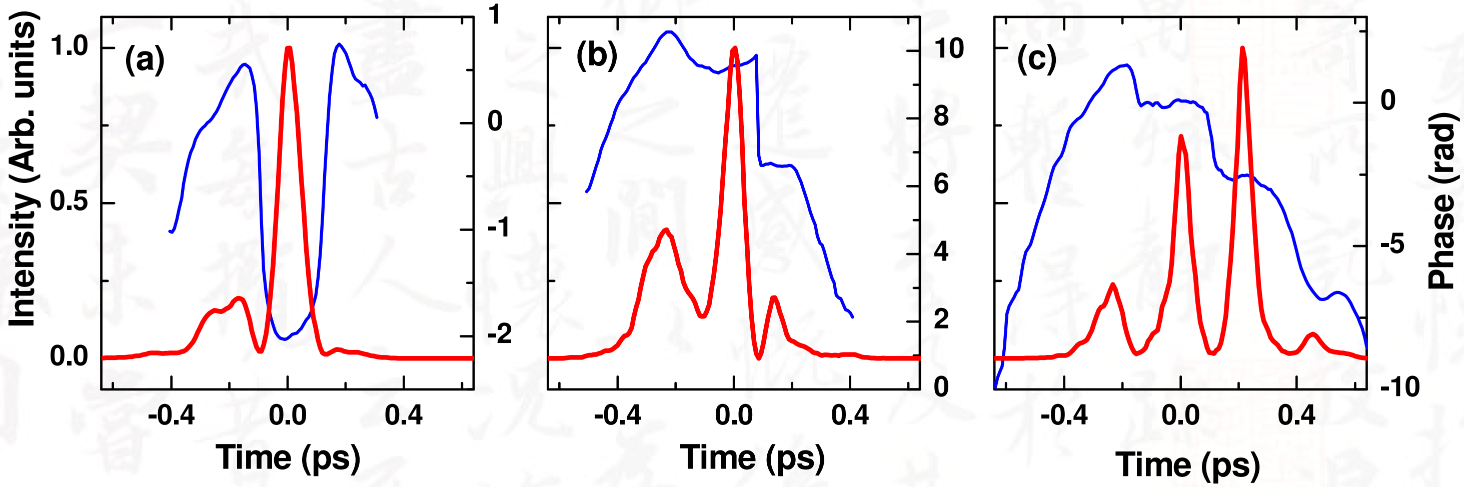


6 Hz, 0.5 ps, 50 μJ @ 120-530 nm
Milton *et al.*, *Science* 292, 2037 (2001)

But more is needed……

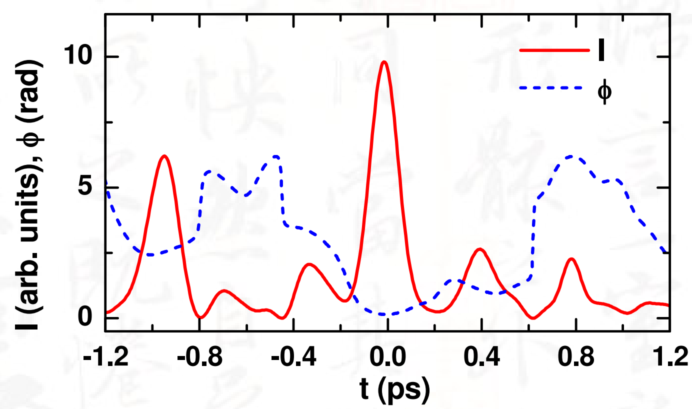


Simulations



Ginger simulation

Simple simulation



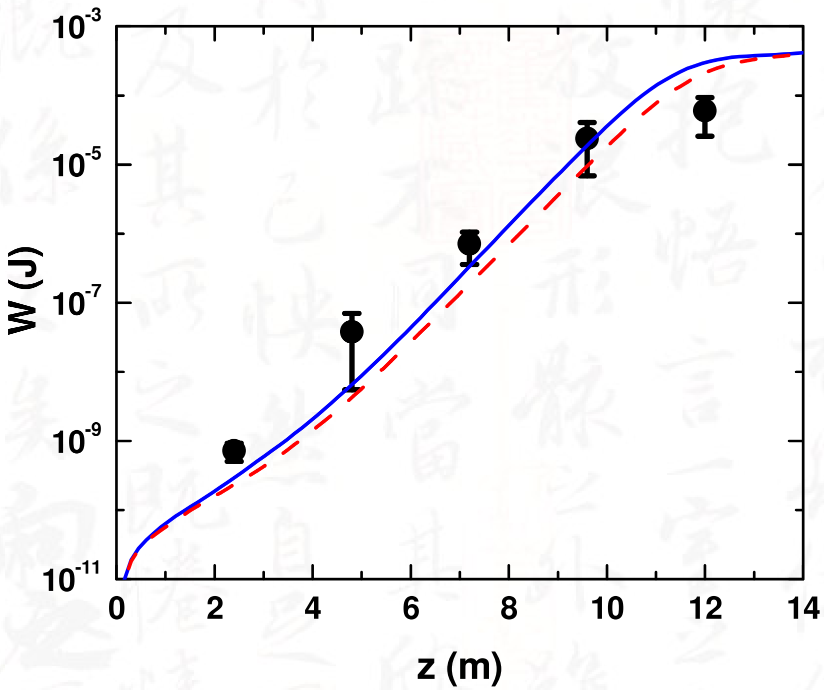
Outline

- **Experiments**
  Frequency-resolved optical gating-FROG
- **Temporal structure and phase**
- **Spectral statistics**
  Correlation to temporal structure
- **Conclusion**

Experiment conditions

Table 1: Experimental Parameters

Experiment	A	B
Peak current	850 A	530 A
Effective bunch length (σ_z)	0.5 ps	0.13 ps
Energy chirp (σ_δ/σ_z)	28 m^{-1}	65 m^{-1}
rms normalized emittance	9 π μm	6 π μm
Undulator period (λ_u)	3.3 cm	
Undulator length (each)	2.4 m	
Undulator parameter (K)	3.1	
Beam energy (γmc^2)	217 MeV	
Nominal wavelength (λ)	530 nm	
Repetition rate	6 Hz	
Gain length (L_G)	0.68 m	0.87 m



The single-shot FROG technique

= Frequency Resolved Optical Gating

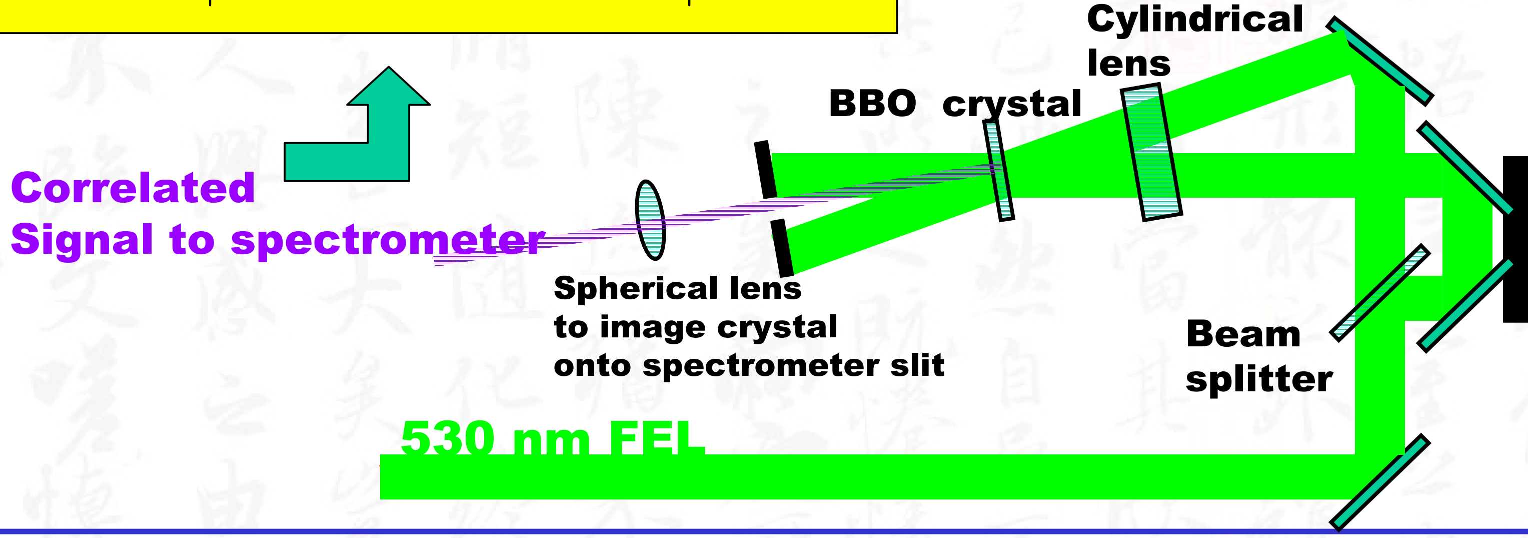
Kane and Trebino, JQE, 29, 571 (1993).
DeLong and Trebino, JOSA B, 11, 2206 (1994).

For the second harmonic FROG

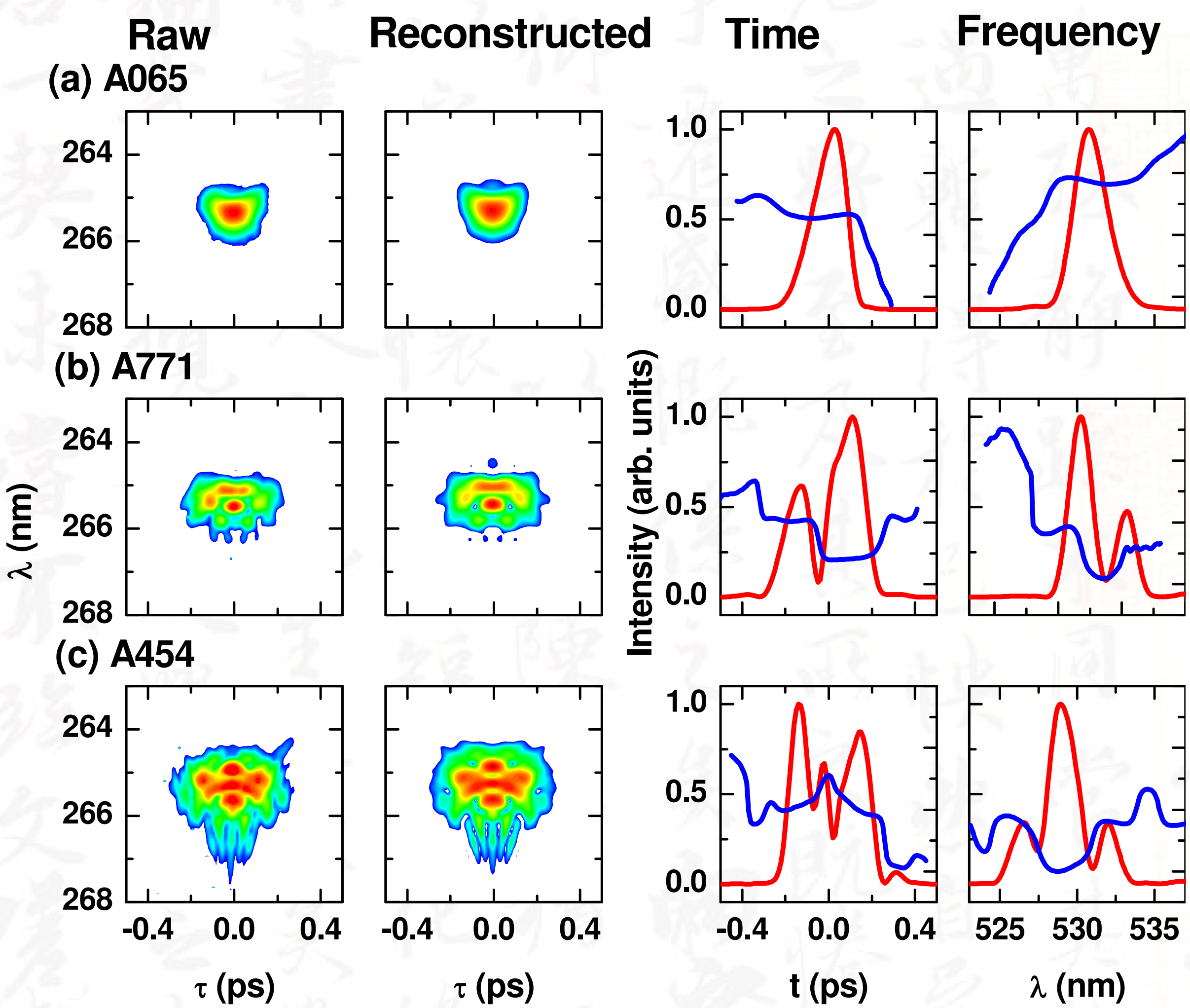
$$E_{sig}(t,\tau) \propto E(t)E(t-\tau).$$

And the measured signal on the spectrometer is

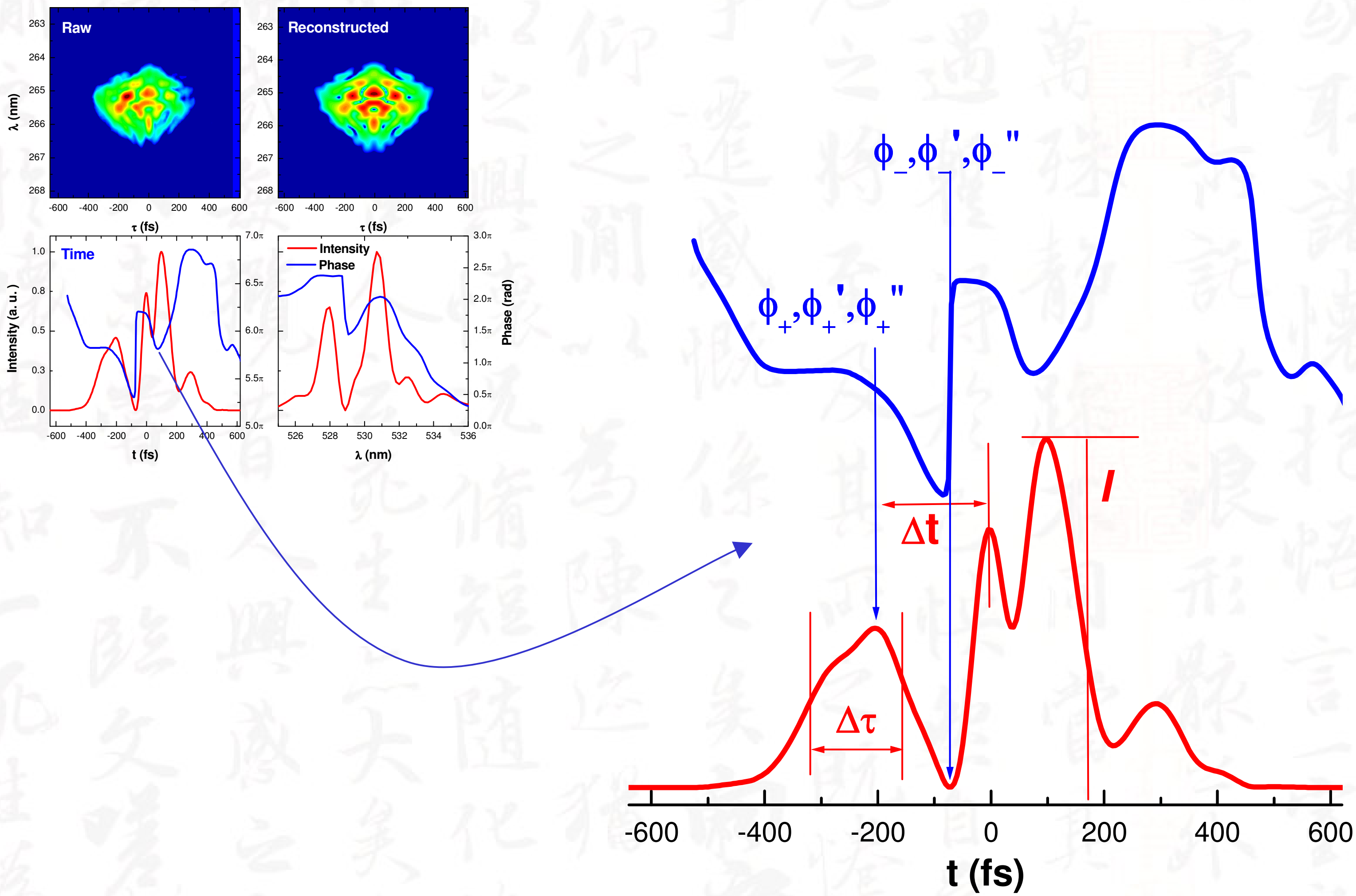
$$I_{FROG}(\omega,\tau) \propto \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)dt \right|^2 .$$



Sample traces



Interest in temporal structure



Temporal structure: Analysis

The field of a SASE FEL (by solving Green's function) is

$$E(t,z) = E_0(z)\sum_{j=i}^{N_e}\exp\left[i\omega_0[1+c\frac{\sigma_\delta}{\sigma_z}(t-t_0)](t-t_j) - \frac{(t-t_j-z/v_g)^2}{4\sigma_t^2}(1-\frac{i}{\sqrt{3}})\right].$$

[S. Krinsky and Z. Huang, Phys. Rev. ST Accel. Beams 6, 050702 (2003).]

Which can be rewritten as

$$E(t) = R(t)\exp[i\phi(t)],$$

where R (normal) and ϕ (uniform) are independent random variables following the distribution

$$\frac{d\phi}{2\pi}\frac{RdR}{\psi_0}e^{-\frac{R^2}{2\psi_0}}.$$

ω_0	resonant frequency	$\omega_0 = \frac{4\pi c\gamma_0^2}{\lambda_u(1+K^2/2)}$
σ_t	coherence length	$\sigma_t = \frac{1}{2\omega_0}\sqrt{\frac{z}{\rho\lambda_u}} \propto n_e^{-1/4}$
σ_δ/σ_z	electron beam energy chirp	

Construction of the probability distributions

Furthermore, n random variables $U_1, U_2, \ldots, U_n$ are said to be *jointly Gaussian* if their joint characteristic function is of the form

$$\mathbf{M}_U(\underline{\omega}) = \exp\left\{ j\,\underline{\bar{u}}^t\underline{\omega} - \tfrac{1}{2}\underline{\omega}^t\underline{C}\,\underline{\omega}\right\} \qquad (2.7\text{-}4)$$

where

$$\underline{\bar{u}} = \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \vdots \\ \bar{u}_n \end{bmatrix}, \qquad \underline{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix} \qquad (2.7\text{-}5)$$

and $\underline{C}$ is an $n \times n$ covariance matrix, with element σ_{ik}^2 in the ith row and kth column defined by

$$\sigma_{ik}^2 = E\left[(u_i - \bar{u}_i)(u_k - \bar{u}_k)\right]. \qquad (2.7\text{-}6)$$

The corresponding nth-order probability density function can be shown to be

$$p_U(\underline{u}) = \frac{1}{(2\pi)^{n/2}|\underline{C}|^{1/2}}\exp\left\{-\frac{1}{2}(\underline{u} - \underline{\bar{u}})^t\underline{C}^{-1}(\underline{u} - \underline{\bar{u}})\right\} \qquad (2.7\text{-}7)$$

where $|\underline{C}|$ and $\underline{C}^{-1}$ are the determinant and matrix inverse of $\underline{C}$, respectively, and $\underline{u}$ is a column matrix of the u values.

Goodman, Statistical Optics, (John Wiley & Sons, New York, 1985), p. 35.

Temporal structure: Analysis

After lengthy calculation, one can obtain the distribution function ….. (carried out by S Krinsky)

$\Phi(R, R', R'', \phi, \phi', \phi'')$

Spike width $\xi = \Delta\tau / \langle \Delta\tau \rangle$ distribution

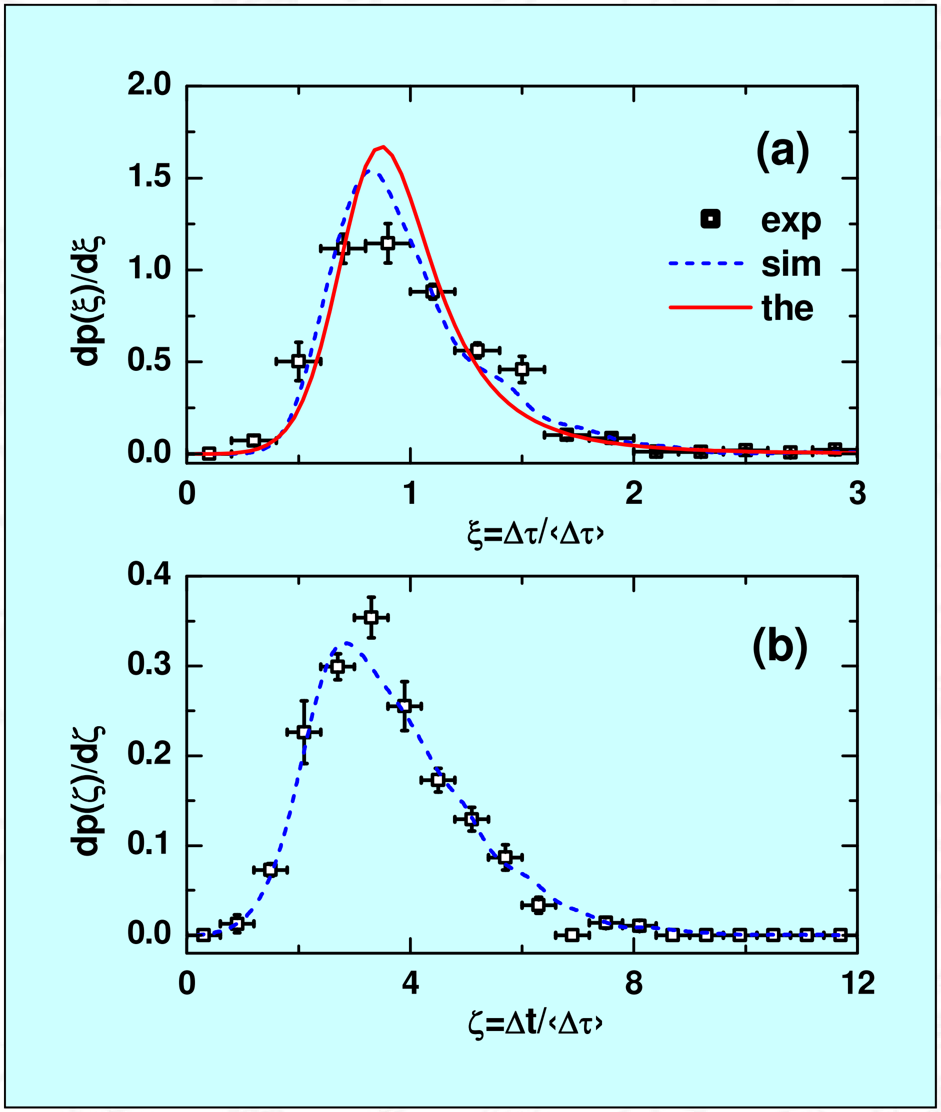
$$\frac{dP(\xi)}{d\xi} = \frac{a\eta}{(a\xi)^5} \int_0^\infty \frac{d\nu}{[3 - 2/(a\xi)^2 + (1/(a\xi)^2 + \nu^2)^2]^{5/2}}.$$

Phase $\nu = \phi' / \sigma_\omega$ distribution at spike maxima (+) and minima (-)

$$\frac{dp_\pm(\nu)}{d\nu} = \frac{\chi}{\sqrt{3+\nu^4}\,[\sqrt{3+\nu^4} \pm (\nu^2 - 1)]^2}.$$

The constants are a=0.8685, η=9.510, χ=0.7925.

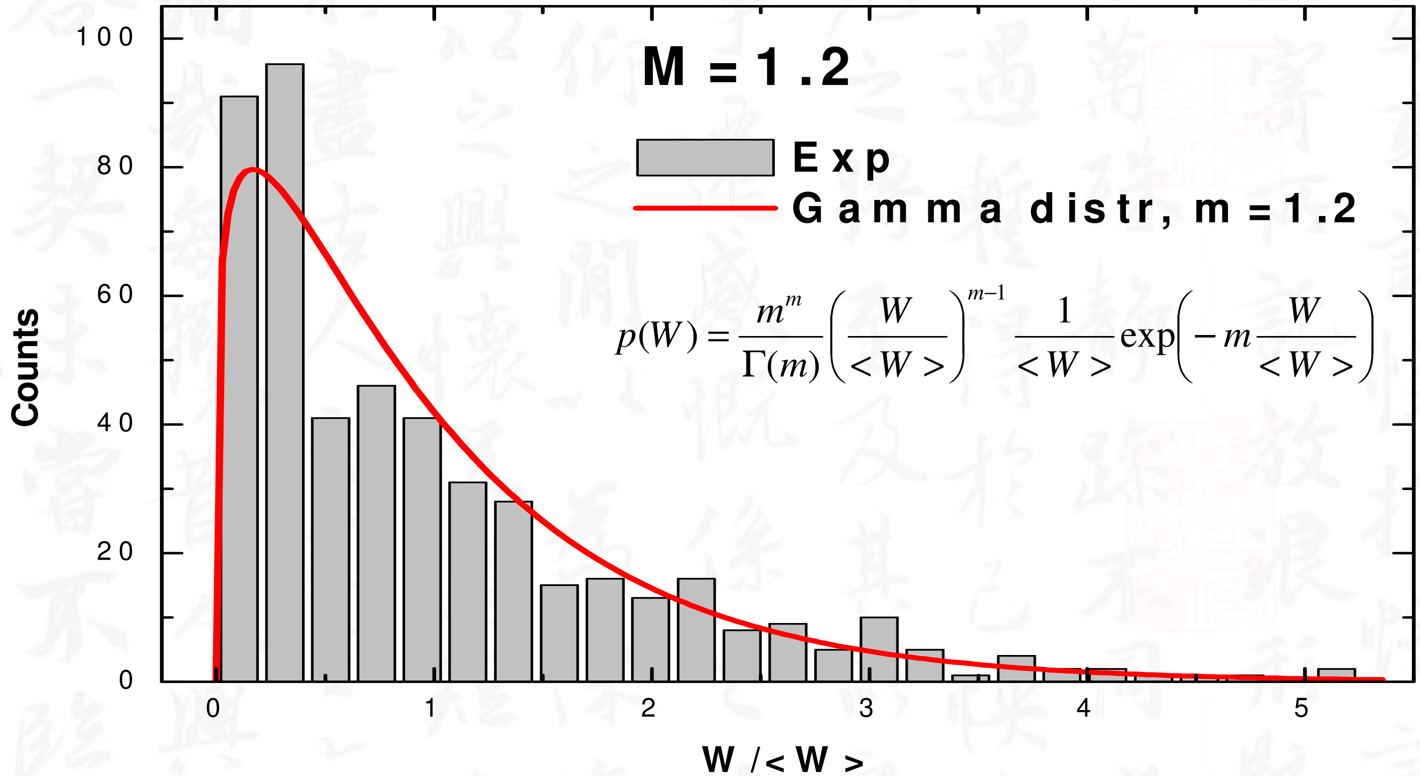
Temporal structure: spike width and spacing



$\langle \Delta\tau \rangle = 52$ fs

Li et al., Phys. Rev. Lett., in press.

What is commonly known now?



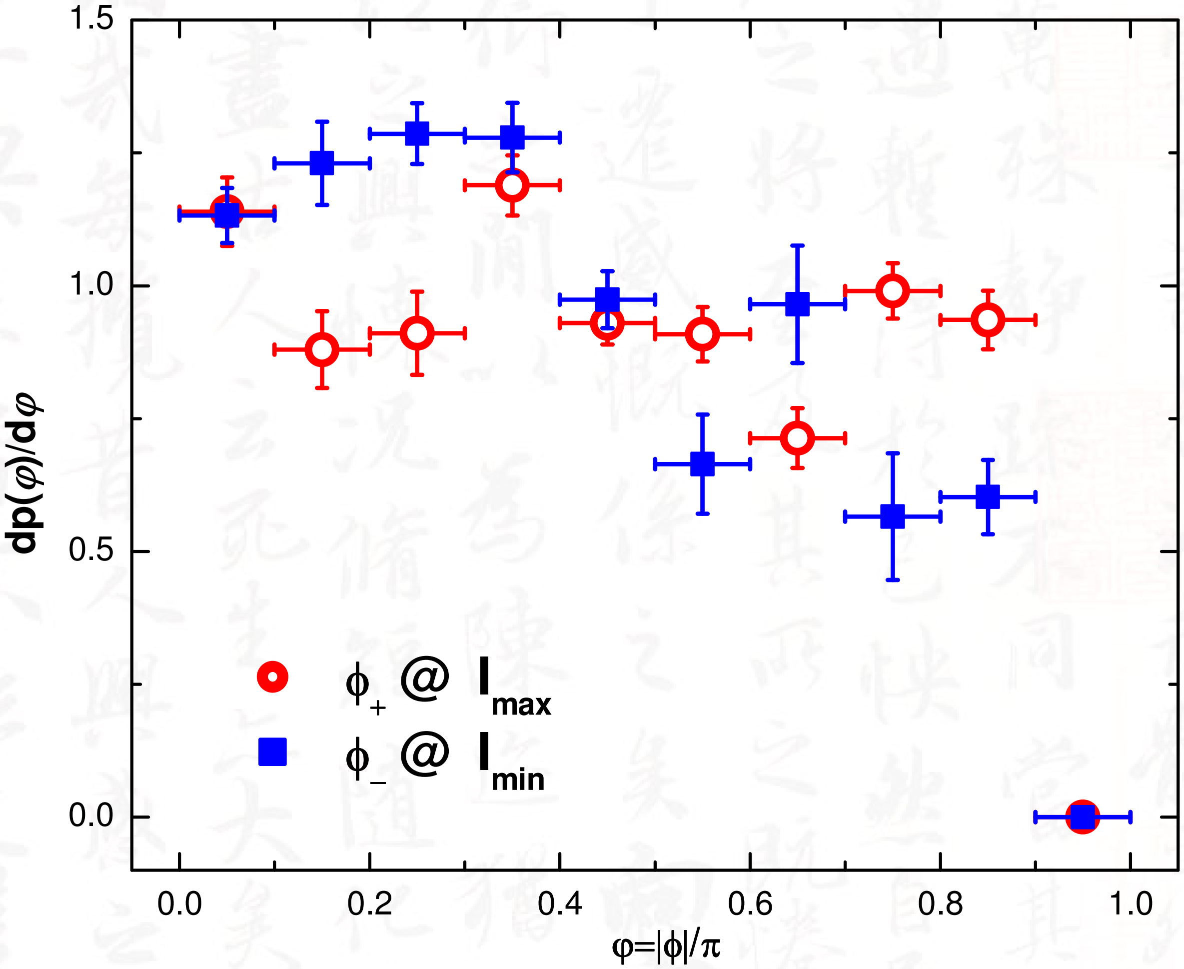
The intensity follows the γ-distribution, as expected.

For similar measurements, see

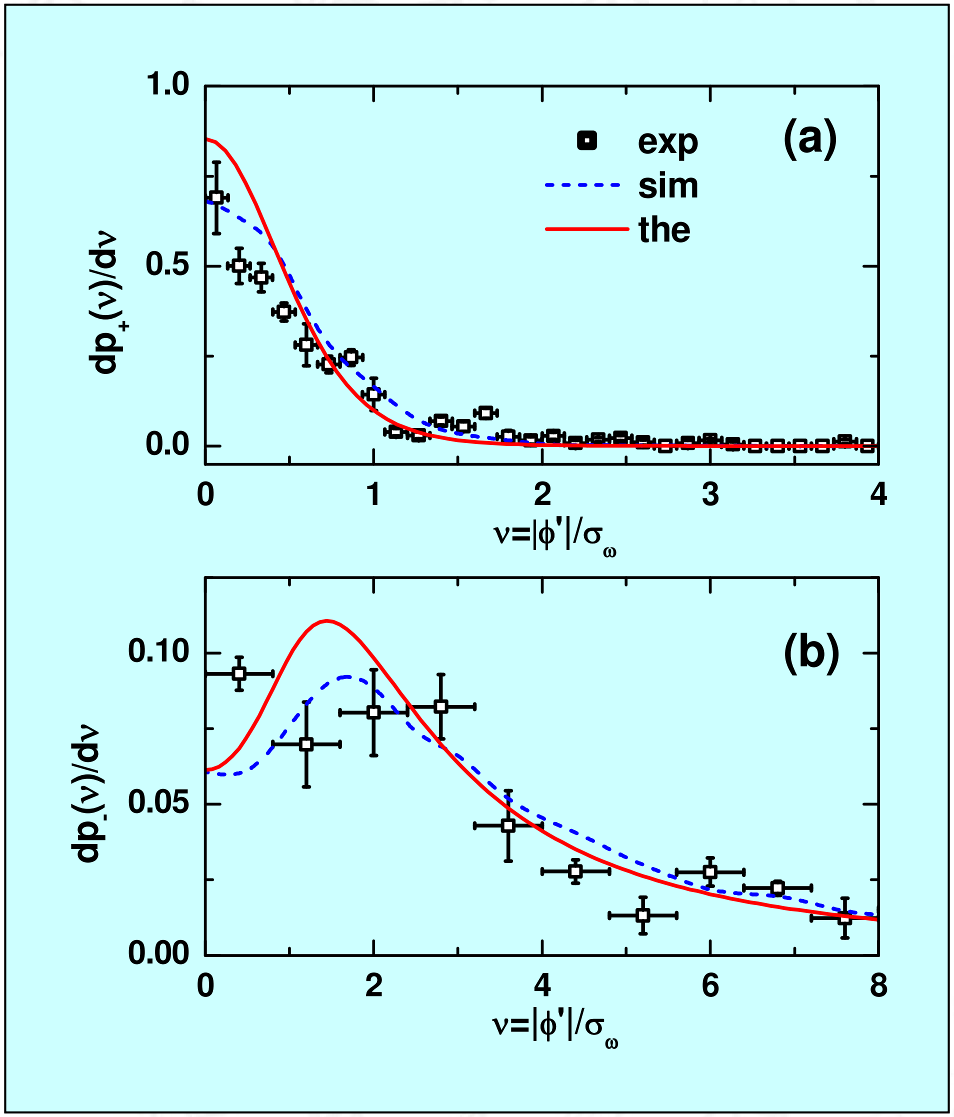
M. Hogan et al., Phys. Rev. Lett. 80, 289 (1998).

M.V. Yurkov, Nucl. Instrum. Methods Phys. Res. A 483, 51 (2002).

Uniform distribution of the phase



Derivative of phase (frequency)



σ_ω=0.0094 rad/fs

Li et al., Phys. Rev. Lett., in press.

Second derivative of phase: chirp

The FEL output is

$$E(t,z) = E_o(z)\sum_{j=i}^{N_e}\exp\left[i\omega_0[1 + c\frac{\sigma_\delta}{\sigma_z}(t-t_0)](t-t_j) - \frac{(t-t_j-z/v_g)^2}{4\sigma_t^2}(1-\frac{i}{\sqrt{3}}) \right].$$

Beam chirp **Intrinsic chirp**

The total chirp in the pulse is

$$\phi'' = \frac{2}{4\sigma_t^2\sqrt{3}} + \omega_0 c\frac{2\sigma_\delta}{\sigma_z}.$$

Taking into account the propagation,

$$\phi'' = \frac{d^2\phi}{dt^2} = 2\frac{\Omega\Theta + \Phi''_m}{\Omega^2 + \Phi''^{\,2}_m},$$

where

$$\Theta = \frac{1}{\sqrt{3}} + 4\sigma_t^2\omega_0 c\frac{\sigma_\delta}{\sigma_z},$$

$$\Omega = 4\sigma_t^2 + \Phi''_m.$$

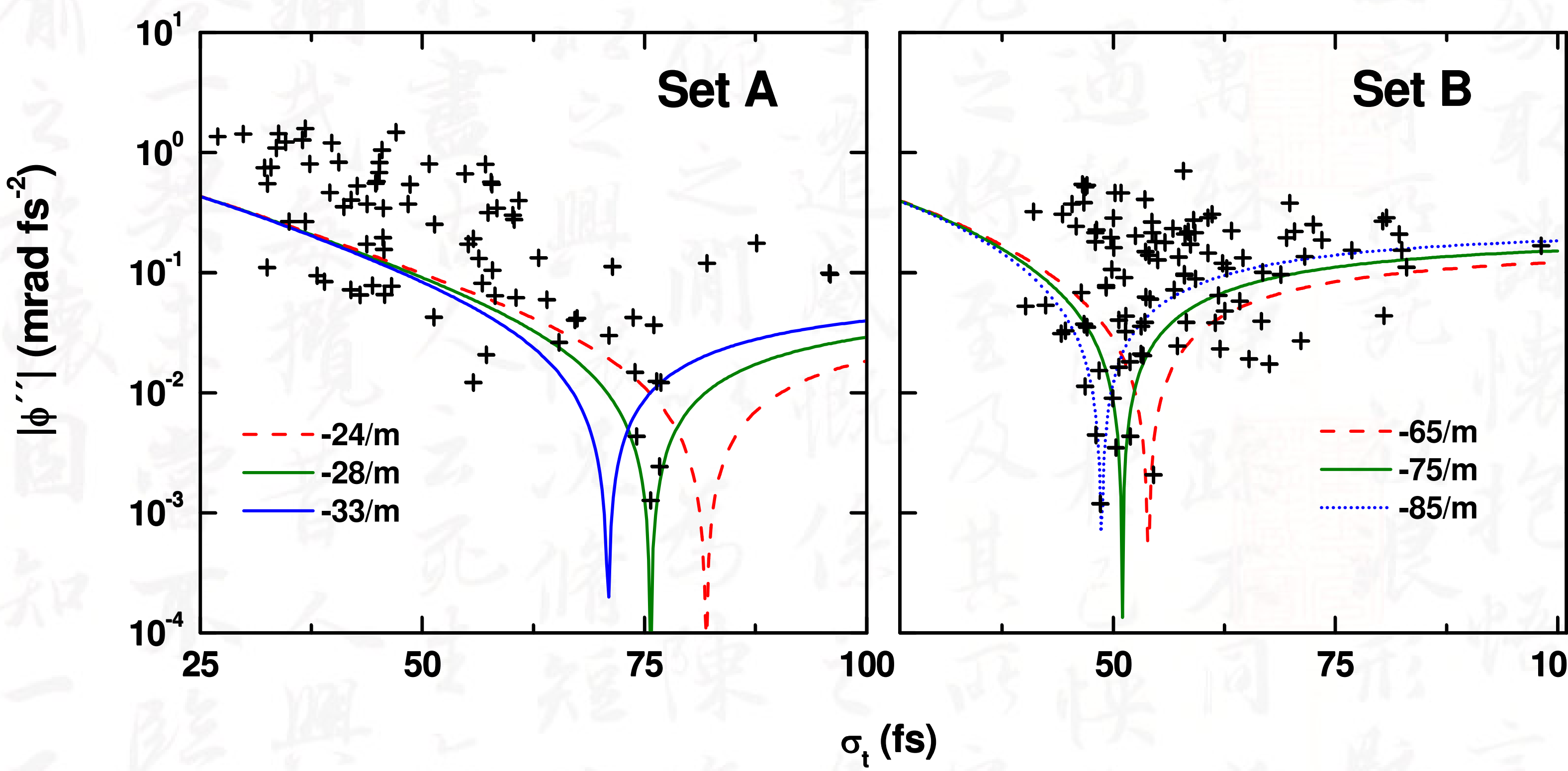
ω_0= resonant frequency $\omega_0 = \frac{4\pi c\gamma_0^2}{\lambda_u(1+K^2/2)}$

σ_t= coherence length $\sigma_t = \frac{1}{2\omega_0}\sqrt{\frac{z}{\rho\lambda_u}} \propto n_e^{-1/4}$

Φ_m''=group velocity dispersion in optics

σ_δ/σ_z=electron beam energy chirp

Chirp analysis: results



$$\phi'' = \frac{d^2\phi}{dt^2} = 2\frac{\Omega\Theta + \Phi_m''}{\Omega^2 + \Phi_m''^{\,2}},$$

[Li et al., Phys. Rev. Lett. **89**, 234801 (2002); **90**,199903 (2003)]

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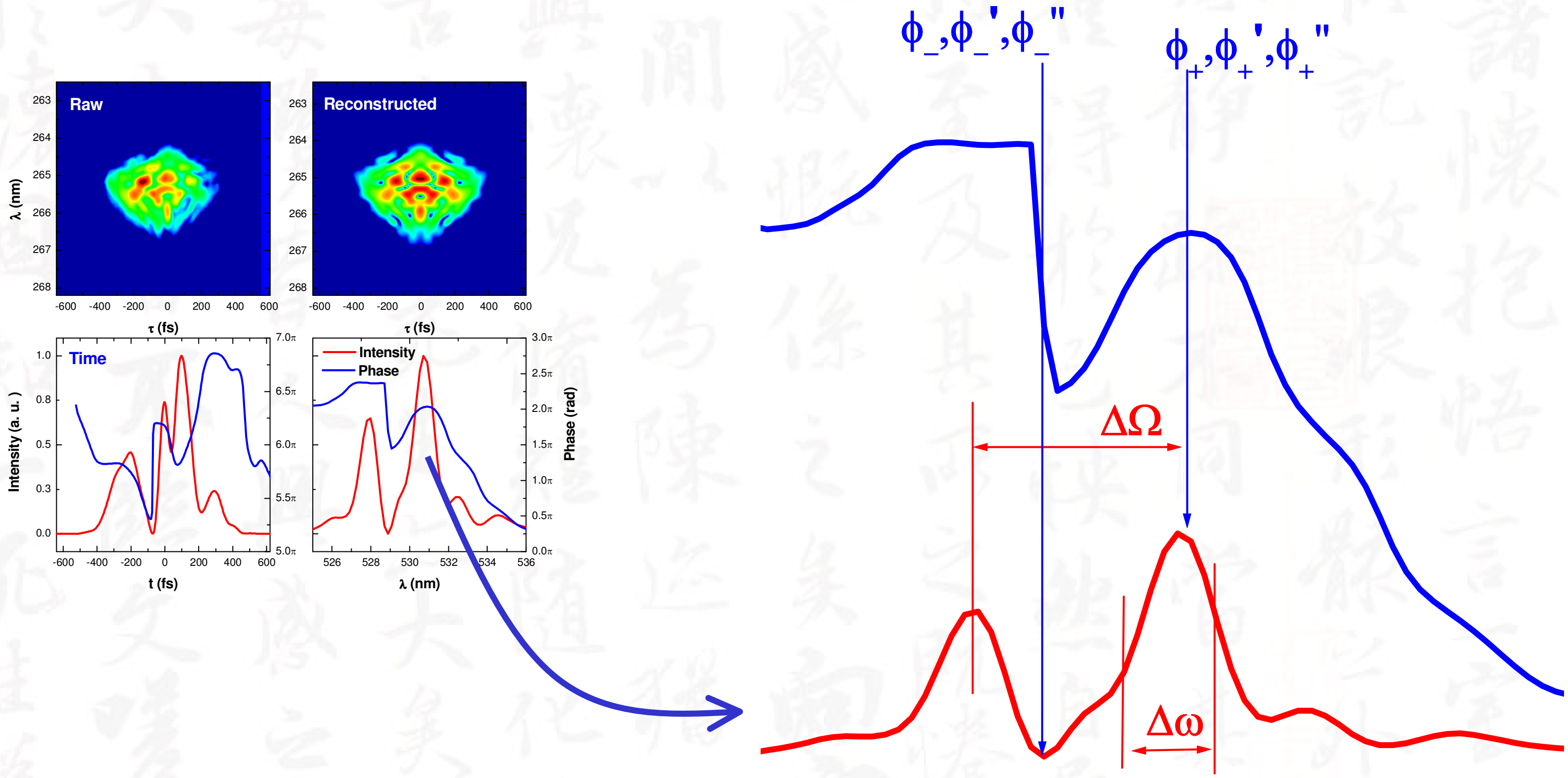
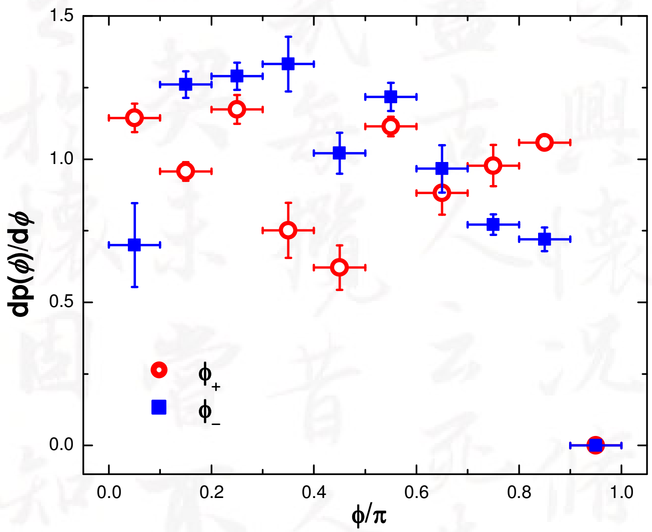
Frequency domain

$$E(\omega, z) \propto \exp\left[-\frac{(\omega-\omega_0)^2}{4\sigma_\omega^2}\right]\sum_{j=i}^{N_e}\exp\left[-i(\omega-\omega_0)t_j\right], \quad \sigma_\omega = \frac{1}{2\sigma_t}.$$

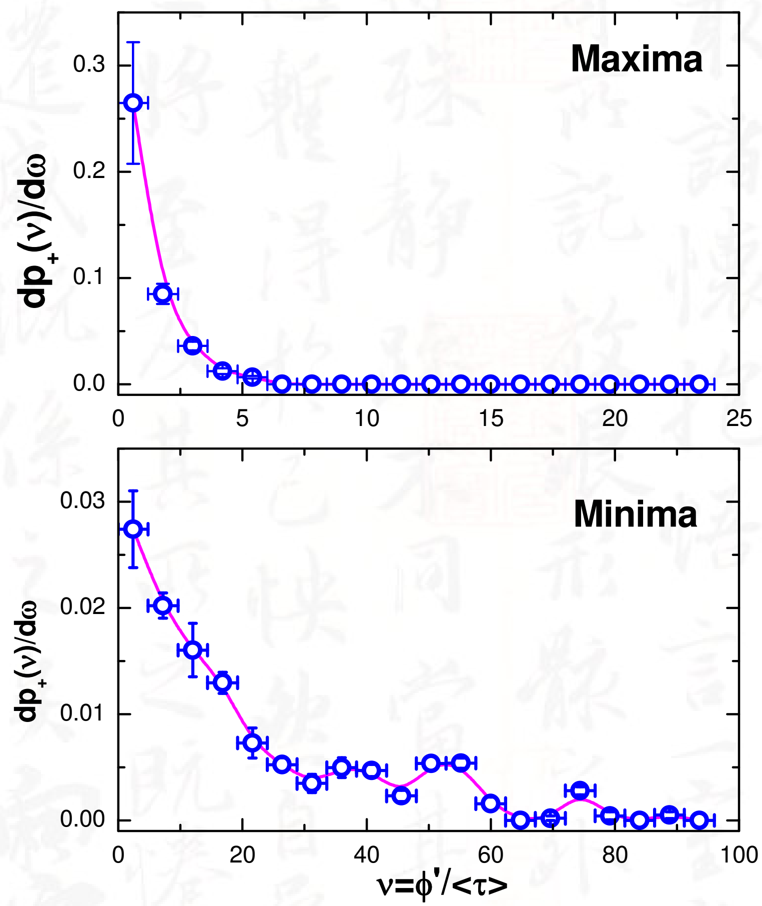


Frequency domain statistics

Phase



Phase derivative



Frequency domain statistics: analysis

- **Time-resolved measurement is not always available, but spectra are easy to measure.**

- **So can one get the time info from spectra?**

In the time domain, the number of coherence modes (coherent spikes) is easily calculated
$\mathbf{m_{time}=T/4\sigma_t}$.

In the frequency domain, the bandwidth is $\sigma_\omega=1/2\sigma_t$ and the spike width is $\Delta\omega=2/T$; therefore

$\mathbf{m_{freq}= \sigma_\omega/ \Delta\omega= T/4\sigma_t}$.

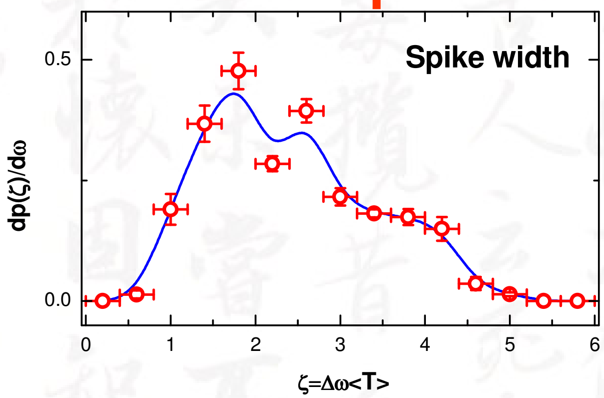
[K.-J. Kim, “Towards X-ray free electron lasers,” ed. R. Bonifacio and W. A. Barletta (American Institute of Physics, New York), p. 3 (1997).]

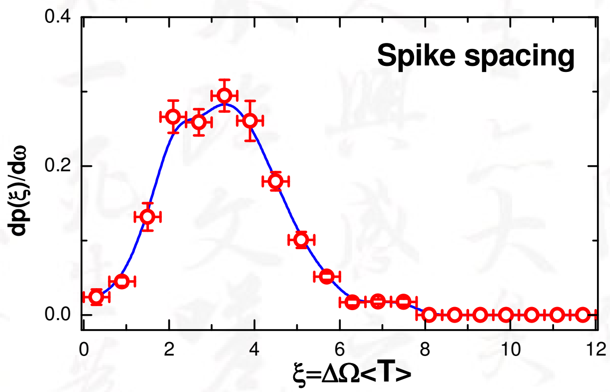
Frequency domain statistics

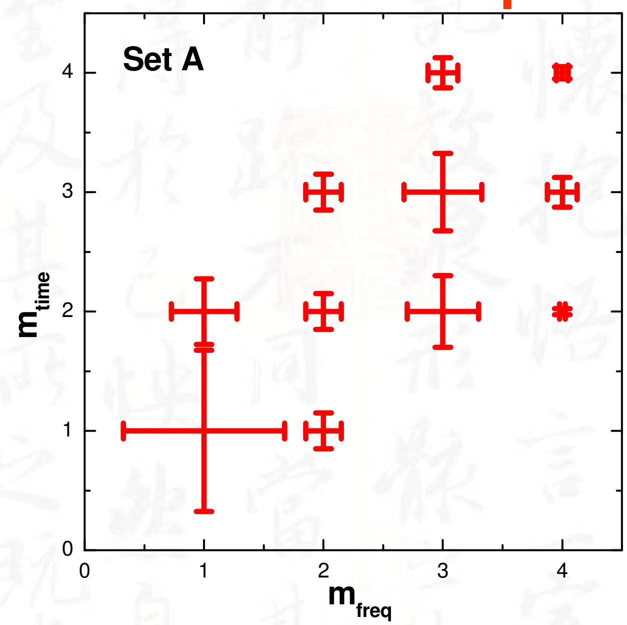
Envelope

$$\langle \Delta\omega T\rangle \approx 2.42 \rightarrow \langle \Delta\omega\rangle \approx 2/T$$
$$\rightarrow m_{freq} \approx \sigma_\omega / \Delta\omega$$
$$\approx (1/2\sigma_t)/(2/T)$$
$$\approx T/4\sigma_t \approx m_{time}$$

$$T = 4m\sigma_t$$
$$T = 2m/\sigma_\omega$$



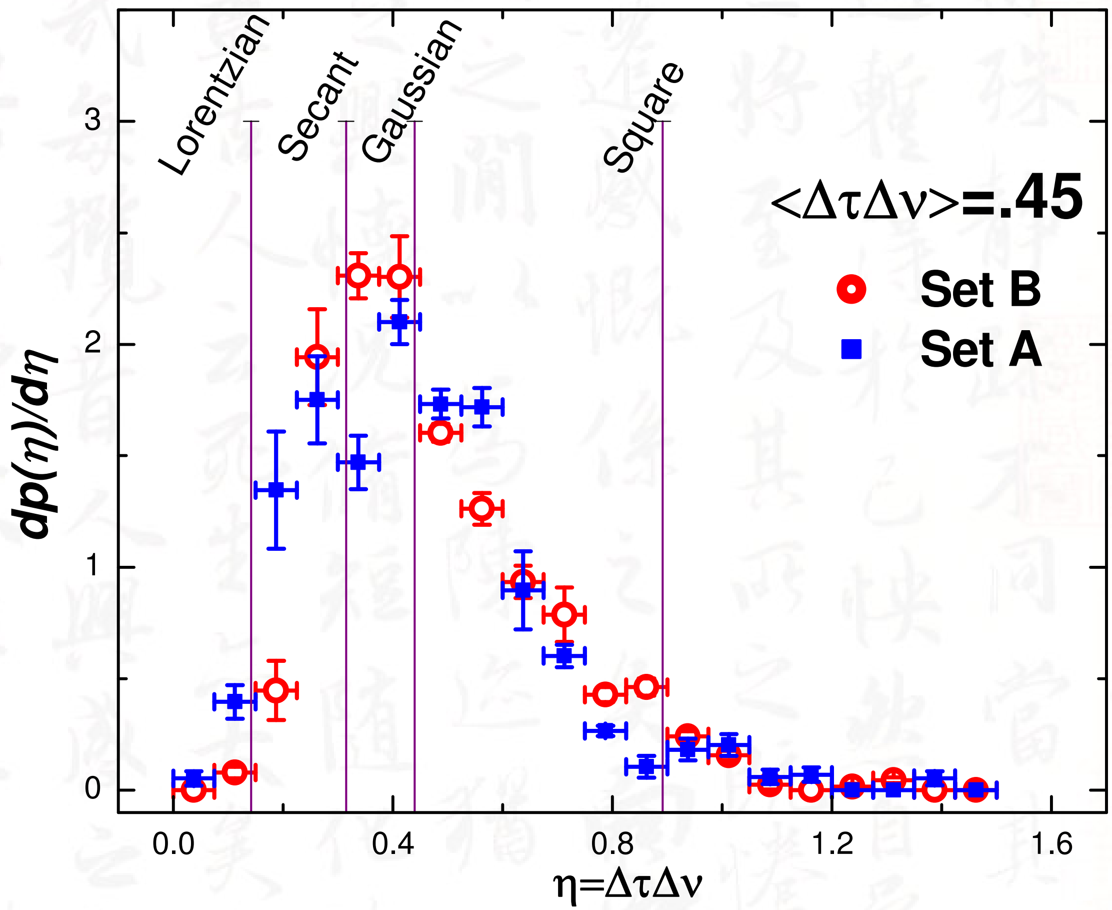




Number of spikes
in time and frequency domain

Time-frequency domain

Time-bandwidth product

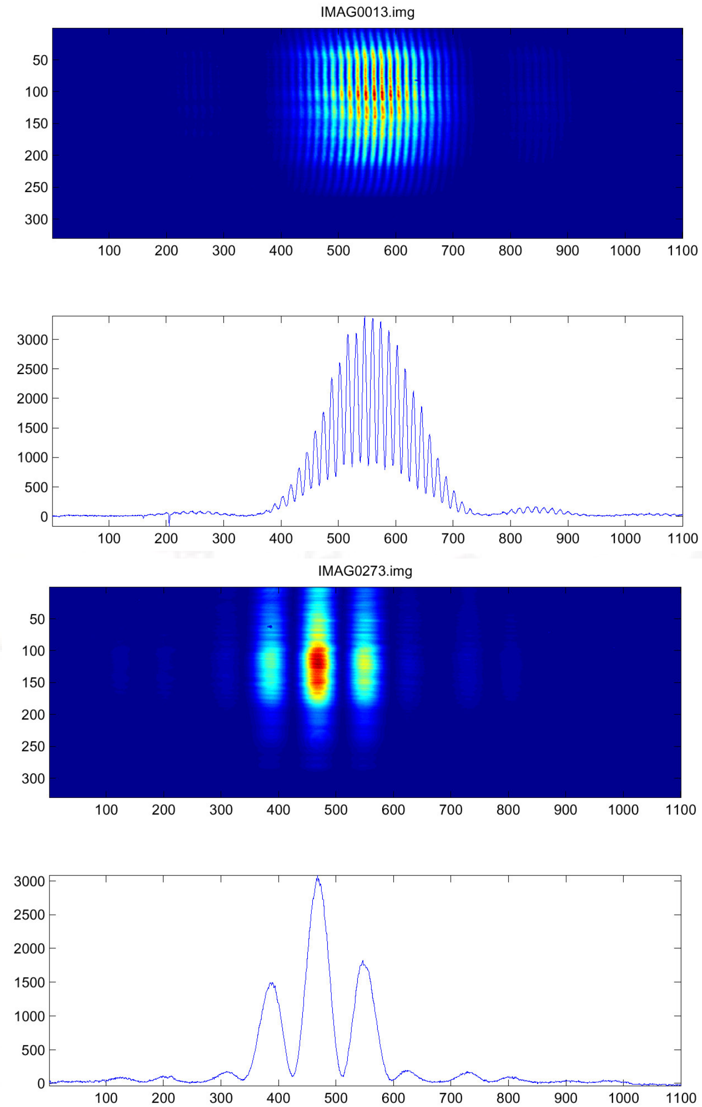


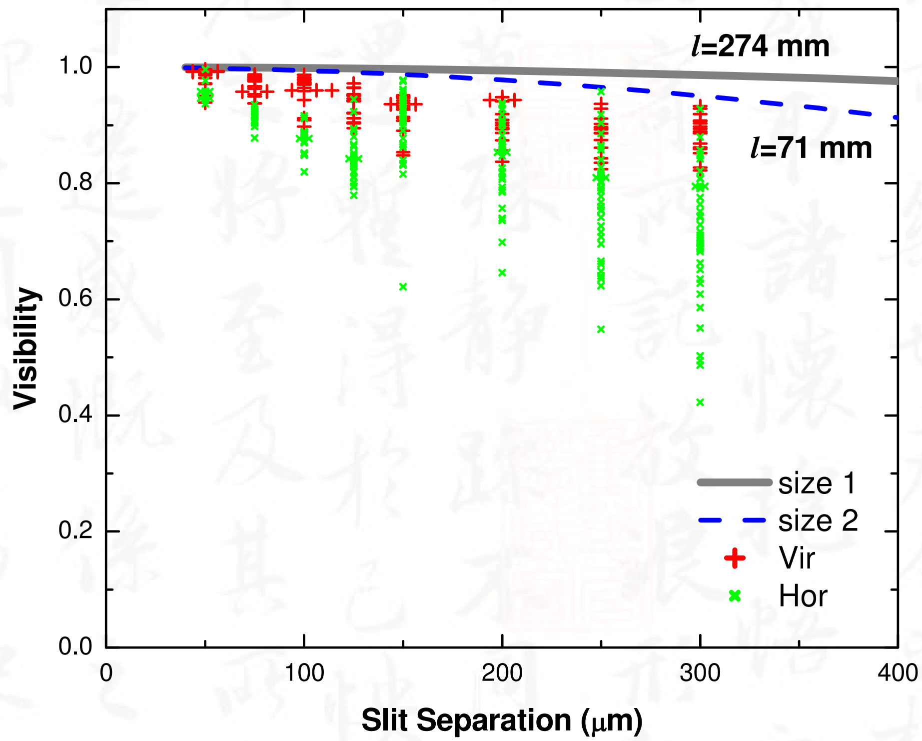
Summary of measurement

Parameters		Data set A	Data set B
General	Gain length L_G (m)	0.68±0.10	0.77±0.07
	Energy per pulse W (μJ)	60	-
	Coherent modes [a]	-	1.2
	rms coherence length σ_t (fs) [b]	53±16	56±10
	Time bandwidth product	0.52±0.31	0.54±0.28
	FWHM pulse duration (fs)	197±105	172±71
	FWHM bandwidth (nm) rms bandwidth (rad/s)	2.6±0.8 $(7.3\pm2.4)\times10^{12}$	3.0±0.9 $(8.5\pm2.5)\times10^{12}$
Spikes: frequency domain	Number of spikes [c]	2.0±1.0	1.3±0.5
	Spike separation $\Delta\lambda$ (nm) [c] $\Delta\omega$ (rad/s)	3.01±1.09 $(20\pm6)\times10^{12}$	3.1±0.9 $(21\pm5)\times10^{12}$
	rms spike width (nm) [d] (rad/s)	0.89±0.34 $(5.9\pm2.3)\times10^{12}$	1.2±0.3 $(7.8\pm1.9)\times10^{12}$
Spikes in time domain	Number of spikes [c]	2.0±1.0	1.3±0.5
	Separation Δt (fs) [c]	175±64	206±61
	Spike length (fs) [d]	53±16	56±10

a. Measured from energy fluctuation
b. Assumed to be similar to spike length, measured only for well-separated spikes
c. Using 0.05 intensity threshold
d. For well-separated spikes only

Spatial coherence: a double slit experiment





A better measurement is needed and is in preparation.

Lin et al., PRL 90, 074801 (2003).

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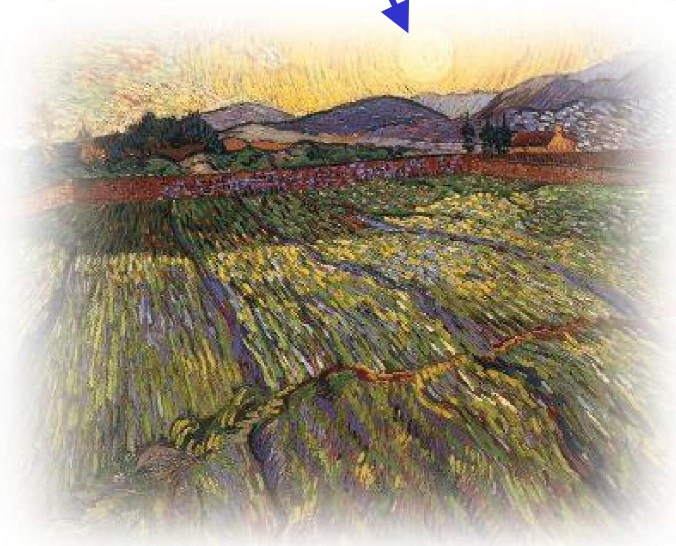
Conclusion

SASE FELs are chaotic light sources……

The sun

Candles + incandescent lights

Stars



but with longer coherence length and a single spatial mode. Each time spike represents a coherence region.

Conclusion

But, one can control the FEL output

by properly controlling the electron beam parameters, including the bunch length and shape, the current, and the correlated energy spread. These translate into control over the FEL output on:

- **Pulse length and temporal structure**
- **Spectral structure**
- **Phase evolution: chirp**
- **Etc……**

To be learned and improved.

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Z. Huang (SLAC)

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