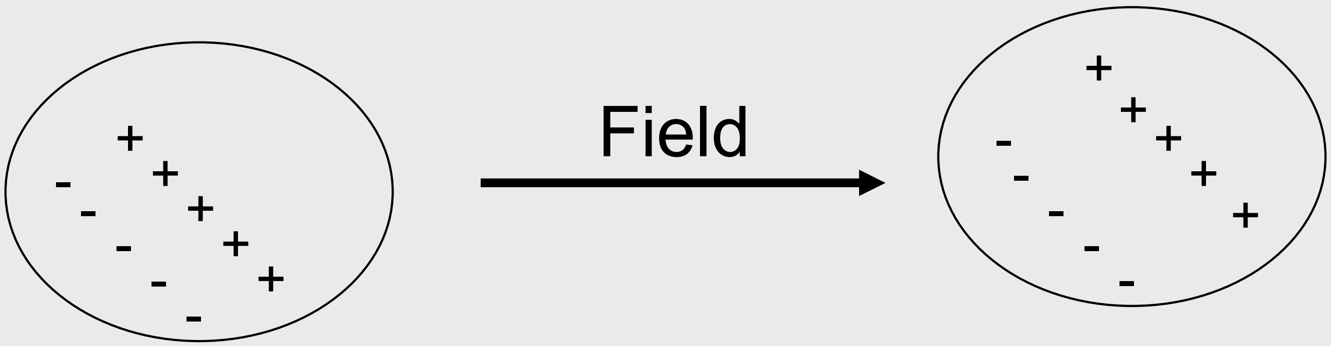
Electro optic effect as e$^-$ beam diagnostics

What is electro-optic effect?



For a nonlinear material, the electric polarization

$$P_i = \varepsilon_0 \sum_{j=1}^{3} \chi_{ij}^{(1)} E_j + \varepsilon_0 \sum_{j=1}^{3}\sum_{k=1}^{3} \chi_{ijk}^{(2)} E_j E_k + \varepsilon_0 \sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3} \chi_{ijkl}^{(3)} E_j E_k E_l + \ldots$$

E_0 is vacuum permittivity, $\chi^{(n)}$ is nth order component of electric susceptibility and is a tensor, I, j, k are cartesian indicesthat run from 1-3, $E_{j\,,k..}$ *can have different frequencies*

For a linear material, the electric polarization higher orders of χ vanish and $P = \varepsilon_0 \chi \cdot E$

The first term $P = \varepsilon_0 \chi^{(1)} : \vec{E}$ applies to all linear optics and yields the common index of refraction and optical dielectric constant

The second term $\varepsilon_0 \sum_{j=1}^{3} \sum_{k=1}^{3} \chi_{ijk}^{2} \vec{E}_j \vec{E}_k$ gives rise to optical mixing ($\omega_1+\omega_2$ and $\omega_1-\omega_2$) and second harmonic generation ($\omega_1=\omega_2$). When one of the fields varies very slowly compared to the other ($\omega_1>>\omega_2$), then it is Pockel's effect where the input and output frequencies are the same and the index of refraction varies linearly with the applied slowly varying field.

$$\vec{P}_i(\omega_1) = \left(\chi_{ijk}^{2} \vec{E}_k(\omega_2)\right)\vec{E}_j(\omega_1)$$

χ^{2}_{ijk} is a 3x3x3 third rank tensor. Since the input and out put frequencies are the same (ω_2~0), it can be contracted notation for (ij) ie. 1=(11), 2=(22), 3=(33), 4=(23), 5=(13), 6= 12). This reduces the number of independent tensor elements from 27 to 18. The crystal symmetry may further reduce it since some of the tensors may be zero. In the contracted form, the 18 elements of χ_{ik} and the components of the polarization vector can be written as

$$\begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix} = \varepsilon_0 \begin{vmatrix} \chi_{11}\chi_{12}\chi_{13}\chi_{14}\chi_{15}\chi_{16} \\ \chi_{21}\chi_{22}\chi_{23}\chi_{24}\chi_{25}\chi_{26} \\ \chi_{31}\chi_{32}\chi_{33}\chi_{34}\chi_{35}\chi_{36} \end{vmatrix} \begin{vmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_yE_z \\ 2E_xE_z \\ 2E_xE_y \end{vmatrix}$$

How to relate to measurable quantity?

The index of refraction n(ω) and the dielectric constant ε (ω) are related to the real part of the susceptibility by

$$n^2_{j,k}(\omega) = \frac{\varepsilon_{j,k}(\omega)}{\varepsilon_0} = 1 + \chi^1_{j,k}(\omega)$$

The susceptibility is related to the electro-optic coefficients $\mathbf{r}_{jk}$ through the optical impermeability B_{jk}

$$B_{jk} \quad = (1/\varepsilon)_{jk} \quad = (1/n^2)_{jk}$$

$$\Delta B_j \quad = r_{jk}E_k \quad = -\frac{2\Delta n_j}{n_j^3}$$

$$\Delta n_j = -\frac{1}{2} n_j^3 r_{jk} E_k$$

Phase difference and intensity modulation are

$$\delta = \frac{2\pi l \Delta n}{\lambda} \qquad I = I_0 \sin^2(\frac{\delta}{2})$$

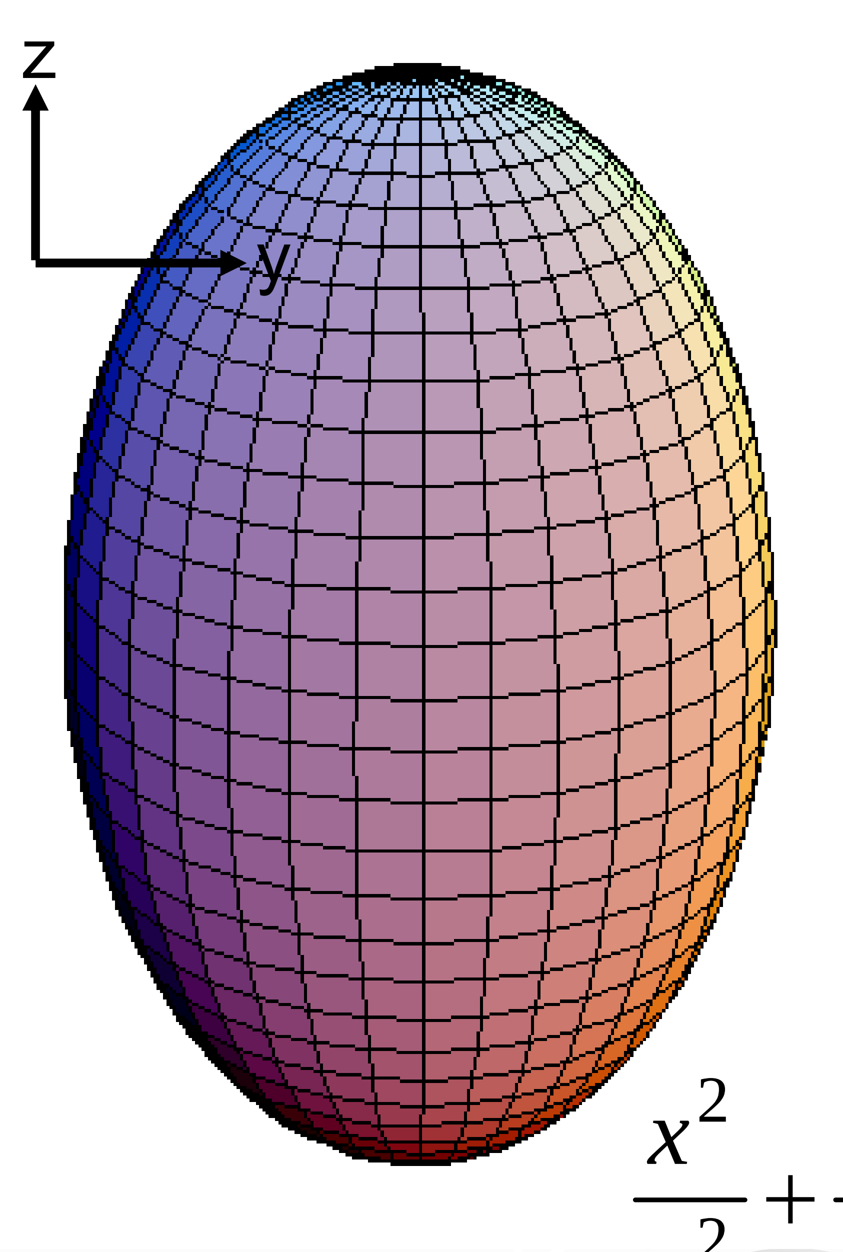
Index of ellipsoid: general

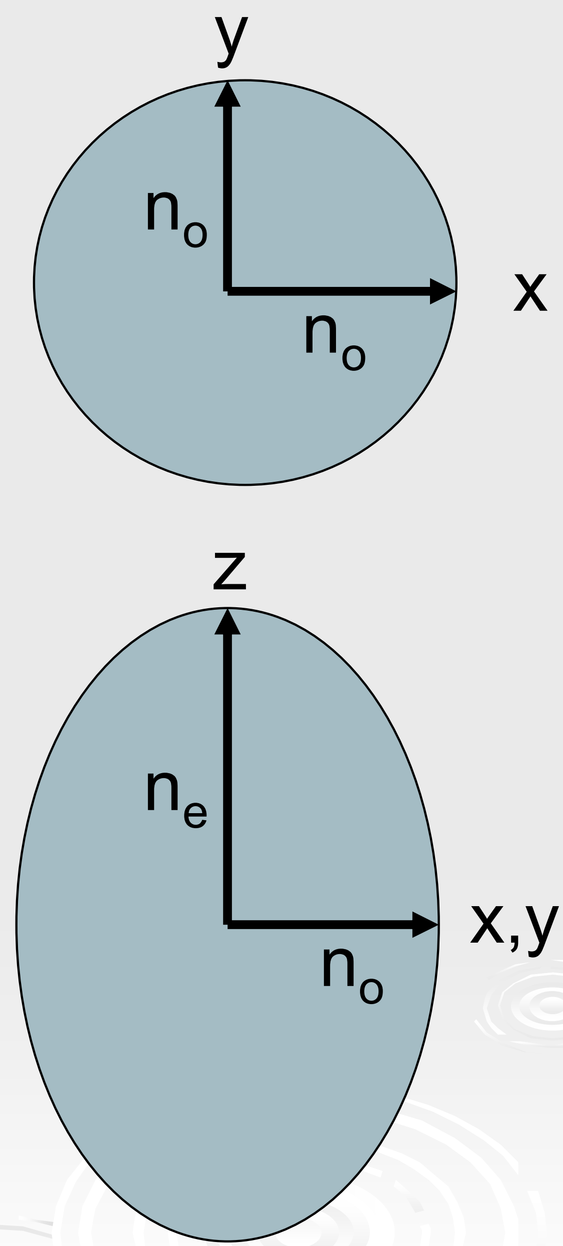
$$\frac{x^2}{N_1^2} + \frac{y^2}{N_2^2} + \frac{z^2}{N_3^2} + \frac{2yz}{N_4^2} + \frac{2xz}{N_5^2} + \frac{2xy}{N_6^2} = 1$$

Choose coordinate system appropriately

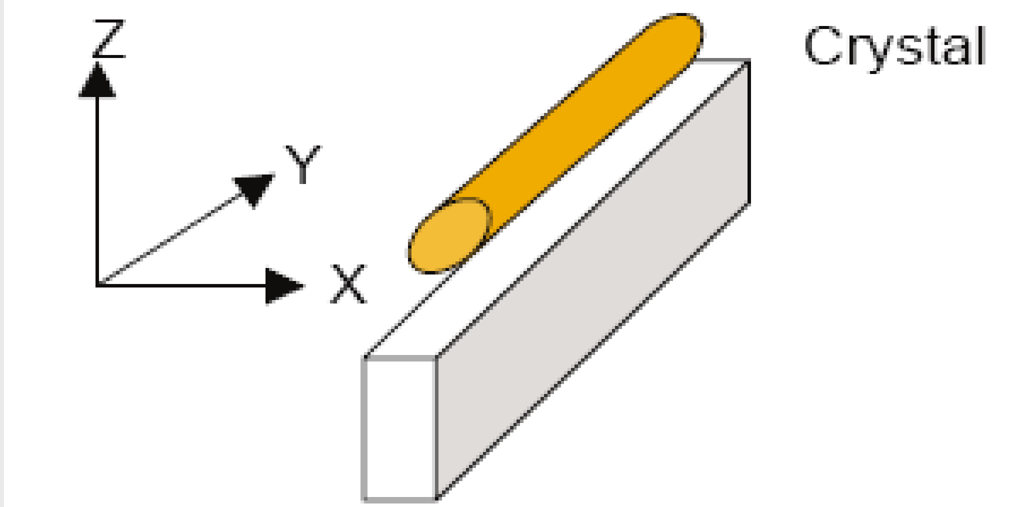
$$\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1$$

Index Ellipsoid: Uniaxial crystal





$$\frac{x^2}{n_0^2}+\frac{y^2}{n_0^2}+\frac{z^2}{n_e^2}=1$$



The electric field dE in the crystal with dielectric constant ε at a distance r from the relativistic electron beam due to a charge σdv

$$d\vec{E} = (\gamma/4\pi\varepsilon_0)\sigma dv/\varepsilon r^2 \vec{r}$$

The field falls off rapidly in the y direction. The dominant field components are

$$dE(x,t) = (\gamma/4\pi\varepsilon_0)\sigma dv/\varepsilon r^2(\bar{x}\cdot\bar{r}),$$

$$dE(z,t) = (\gamma/4\pi\varepsilon_0)\sigma dv/\varepsilon r^2(\bar{z}\cdot\bar{r})$$

Time dependent field leads to time dependent change in index of refraction and index of ellipsoid

$$\Delta n_j = -\frac{1}{2} n_j^3 r_{jk} E_k \qquad \frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1$$

$$x^2\left(\frac{1}{n_1^2} + r_{1j}E_j\right) + y^2\left(\frac{1}{n_2^2} + r_{2j}E_j\right) + z^2\left(\frac{1}{n_3^2} + r_{3j}E_j\right) + 2yz(r_{4j}E_j) + 2xz(r_{5j}E_j) - 2xy(r_{6j}E_j) = 1$$

Redefine the principal axes X_i, calculate the refractive indices N_i and the phase difference δ between the orthogonal components

$$\delta = \frac{2\pi}{\lambda} \int_0^L N_j - N_i dx_k$$

where x_k is the laser propagation direction, L is length of the crystal in that direction and $N_{i,j}$ are refractive indices in directions orthogonal to propagation

$$I = I_o \sin^2(2\theta + 2\phi) \sin^2 \frac{\delta}{2}$$

where θ is the angle made by the E vector of the laser beam with the y-z plane, Φ is the field dependent orientation of the new principal axis with the old one

Calculations for Lithium Niobate

Direction of propagation of charge bunch	Direction of propagation of laser light		
	x	y	z
x	$\phi \sim \frac{1}{2}\tan^{-1}\left(\frac{36.4\times10^{-12}E_y}{-0.016+3.4\times10^{-12}E_y-21.1\times10^{-12}E_z}\right)$	$\phi \sim 0$	$\phi \sim 0$
y	$\phi \sim 0$	$\phi \sim \frac{1}{2}\tan^{-1}\left(\frac{36.4\times10^{-12}E_x}{-0.016-21.1\times10^{-12}E_z}\right)$	$\phi \sim \pm\pi/4$
z	$\phi \sim \frac{1}{2}\tan^{-1}\left(\frac{36.4\times10^{-12}E_y}{-0.016+3.4\times10^{-12}E_y}\right)$	$\phi \sim \frac{1}{2}\tan^{-1}\left(\frac{36.4\times10^{-12}E_x}{-0.016-3.4\times10^{-12}E_y}\right)$	$\phi \sim \frac{1}{2}\tan^{-1}\left(\frac{E_x}{E_y}\right)$

Four configurations have field independent Φ: electrons along x with laser along y and z, electrons along y, laser along x and z

TABLE III. Total retardation δ experienced by the laser beam for three possible propagation directions of the optical and electron beams. It is assumed that the optic axis is along the z direction, and the electric field along the direction of propagation is zero for charge bunches with $\gamma \gg 1$.

Direction of propagation of charge bunch	Direction of propagation of laser light: x	y	z
x	$\delta = -(1.19\times10^{6})L - (0.0014)\int_0^L E_z\,dx + (0.0003)\int_0^L E_y\,dx - (2.97\times10^{-12})\int_0^L E_y^2\,dx + (1.95\times10^{-13})\int_0^L E_z^2\,dx - (1.67\times10^{-14})\int_0^L E_yE_z\,dx + \cdots$	$\delta = -(1.19\times10^{6})L - (0.0014)\int_0^L E_z\,dy + (0.0003)\int_0^L E_y\,dy + (1.95\times10^{-13})\int_0^L E_z^2\,dy - (1.67\times10^{-14})\int_0^L E_yE_z\,dy - (3.69\times10^{-15})\int_0^L E_y^2\,dy + \cdots$	$\delta = -(0.0011)\int_0^L E_y\,dz + (6.69\times10^{-14})\int_0^L E_yE_z\,dz - (3.51\times10^{-24})\int_0^L E_yE_z^2\,dz - (9.13\times10^{-25})\int_0^L E_y^3\,dz + \cdots$
y	$\delta = -(1.19\times10^{6})L - (0.0014)\int_0^L E_z\,dx + (1.95\times10^{-13})\int_0^L E_z^2\,dx - (2.51\times10^{-23})\int_0^L E_z^3\,dx + \cdots$	$\delta = -(1.19\times10^{6})L - (0.0014)\int_0^L E_z\,dy + (2.96\times10^{-12})\int_0^L E_x^2\,dy + (1.95\times10^{-13})\int_0^L E_z^2\,dy + \cdots$	$\delta = -(0.0005)\int_0^L E_x\,dz + (3.34\times10^{-14})\int_0^L E_xE_z\,dz - (1.76\times10^{-24})\int_0^L E_xE_z^2\,dz - (1.14\times10^{-25})\int_0^L E_x^3\,dz + \cdots$
z	$\delta = -(1.19\times10^{6})L + (0.0003)\int_0^L E_y\,dx - (2.97\times10^{-12})\int_0^L E_y^2\,dx - (5.71\times10^{-22})\int_0^L E_y^3\,dx + \cdots$	$\delta = -(1.19\times10^{6})L - (0.0014)\int_0^L E_z\,dy + (0.0003)\int_0^L E_y\,dy - (2.96\times10^{-12})\int_0^L E_x^2\,dy - (5.92\times10^{-12})\int_0^L E_yE_x\,dy - (3.69\times10^{-15})\int_0^L E_y^2\,dy + \cdots$	$\delta = -(0.0005)\int_0^L E_x\,dz - (0.0007)\int_0^L \frac{E_y^2}{E_x}\,dz - (1.14\times10^{-25})\int_0^L E_x^3\,dz + \cdots$

Courtesy: PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME 5, 042801 (2002)

Direction of propagation of charge bunch	x
x	$\delta = -(1.19 \times 10^6)L - (0.0014)\int_0^L E_z\,dx + (0.0003)\int_0^L E_y\,dx - (2.97 \times 10^{-12})\int_0^L E_y^2\,dx + (1.95 \times 10^{-13})\int_0^L E_z^2\,dx - (1.67 \times 10^{-14})\int_0^L E_y E_z\,dx + \cdots$
y	$\delta = -(1.19 \times 10^6)L - (0.0014)\int_0^L E_z\,dx + (1.95 \times 10^{-13})\int_0^L E_z^2\,dx - (2.51 \times 10^{-23})\int_0^L E_z^3\,dx + \cdots$
z	$\delta = -(1.19 \times 10^6)L + (0.0003)\int_0^L E_y\,dx - (2.97 \times 10^{-12})\int_0^L E_y^2\,dx - (5.71 \times 10^{-22})\int_0^L E_y^3\,dx + \cdots$

The magnitude of EO coefficients r_{ij} ~10^{-12}m/V-linear term good approximation

Electron along y, laser along x, the Phase retardation δ is

$$\delta = -(1.19 \times 10^{6})L - (0.0014)\int_0^L E_z\,dx + (1.95 \times 10^{-13})\int_0^L E_z^2\,dx - (2.51 \times 10^{-23})\int_0^L E_z^3\,dx + \cdots$$

Electron along z, laser along x, the Phase retardation δ is

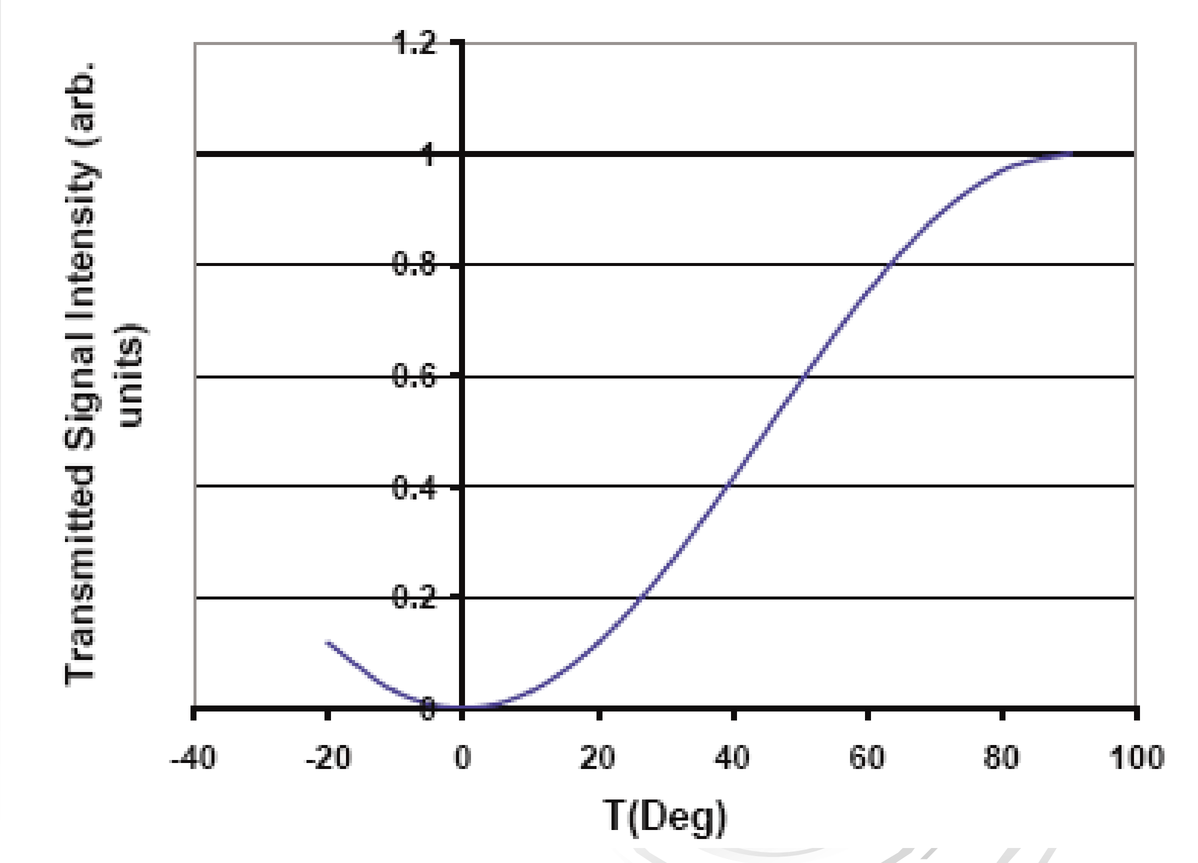
$$\delta = -(1.19 \times 10^{6})L + (0.0003)\int_0^L E_y\,dx - (2.97 \times 10^{-12})\int_0^L E_y^2\,dx - (5.71 \times 10^{-22})\int_0^L E_y^3\,dx + \cdots$$

No cross terms, large coefficient for the first one, large static term

For an electron beam travelling along y axis, laser light along x axis with the polarization of the laser set at 45° to the field free optic axis,

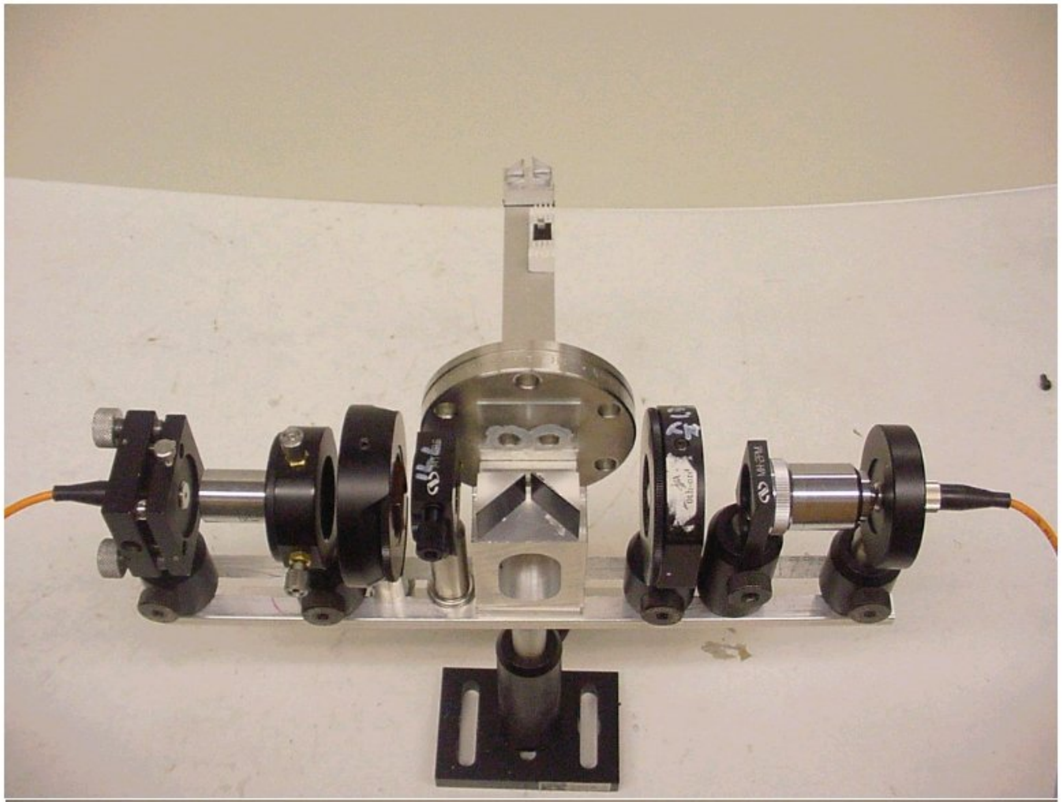
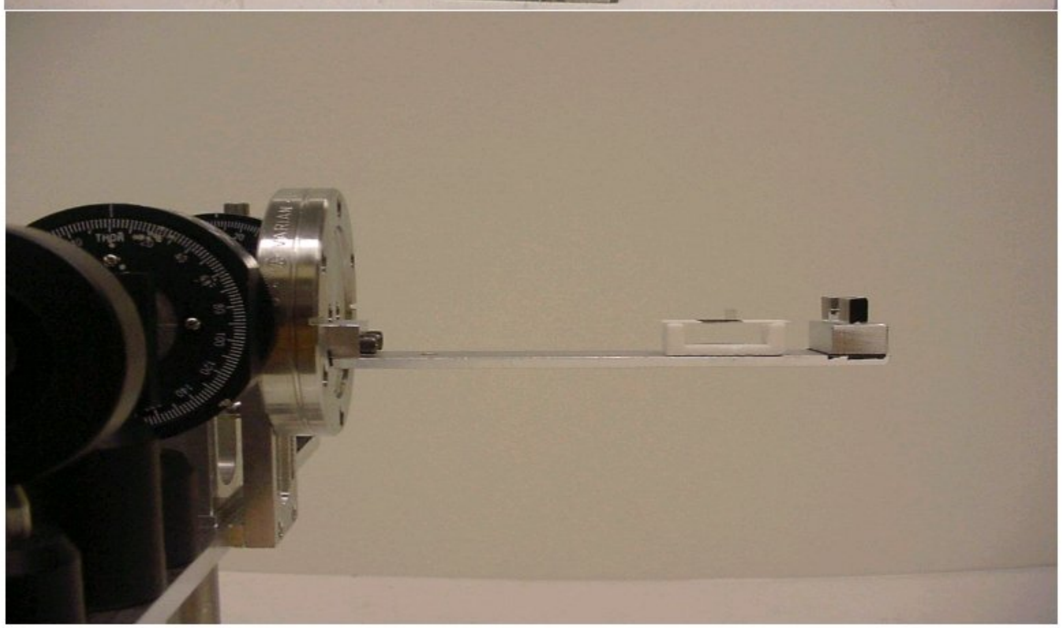
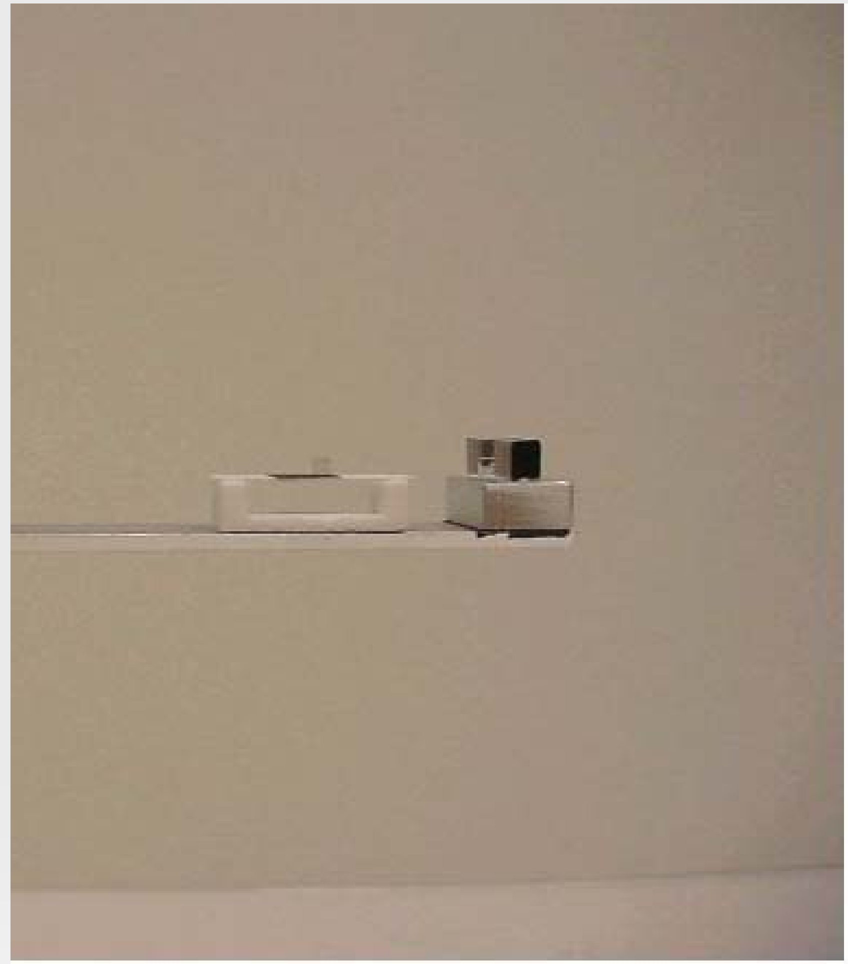
$$I(t) = I_0\{\eta + \sin^2[\delta_b + \delta(t)]\}$$

where η is the extinction coefficient of the optical arrangement



Preset for operation in linear regime

- Select type of Crystal
  - E-O coefficient
  - Radiation damage
- Select orientation
  - Rotation
  - Cross & Static terms
- Select laser
  - Polarization
  - CW/Pulsed
  - Power
  - wavelength
- Select Detection scheme
  - Time
  - Spectrum
  - Spatial profile
- Select operating parameters
  - X'tal dimensions
  - Location
  - Laser transport
  - Optical system
  - Diagnostics
  - ...

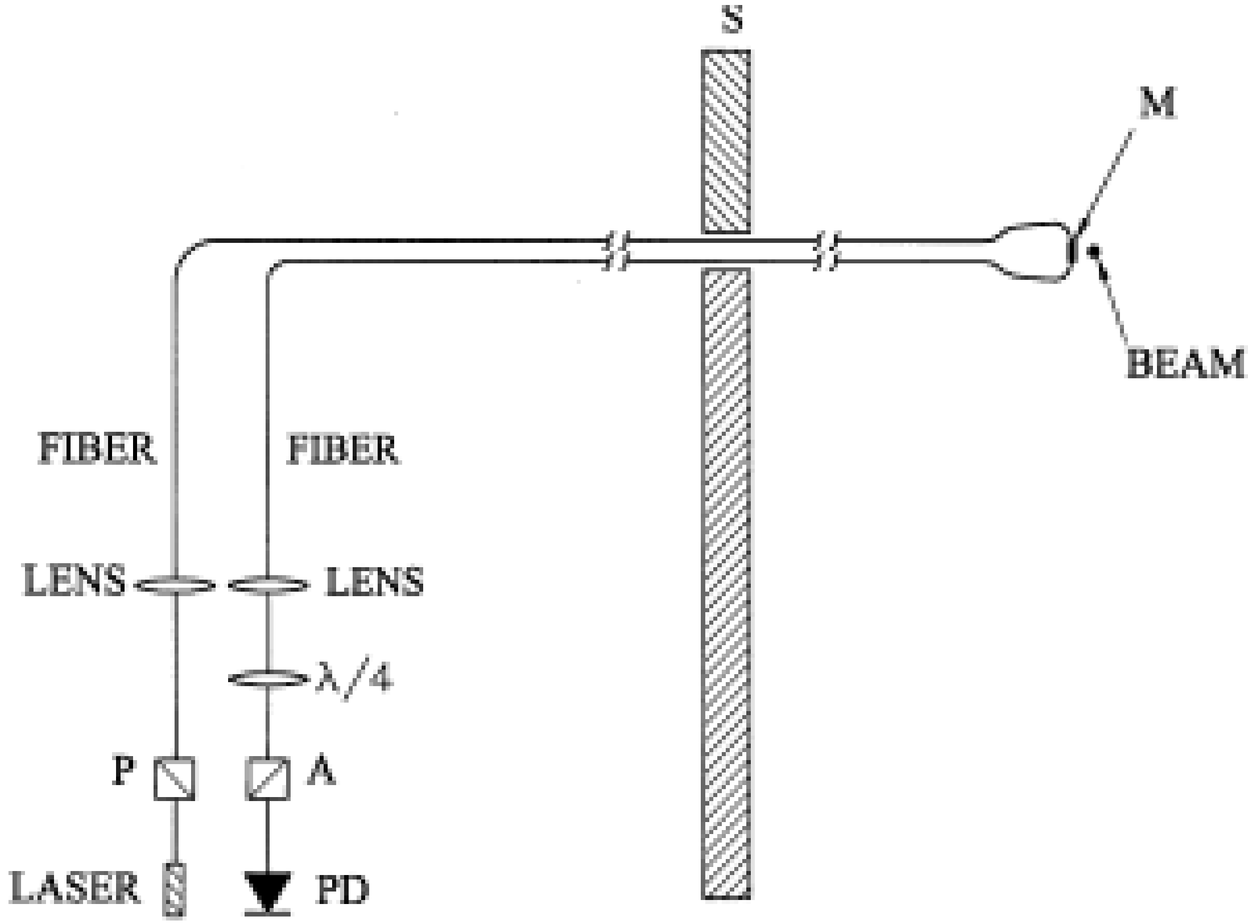


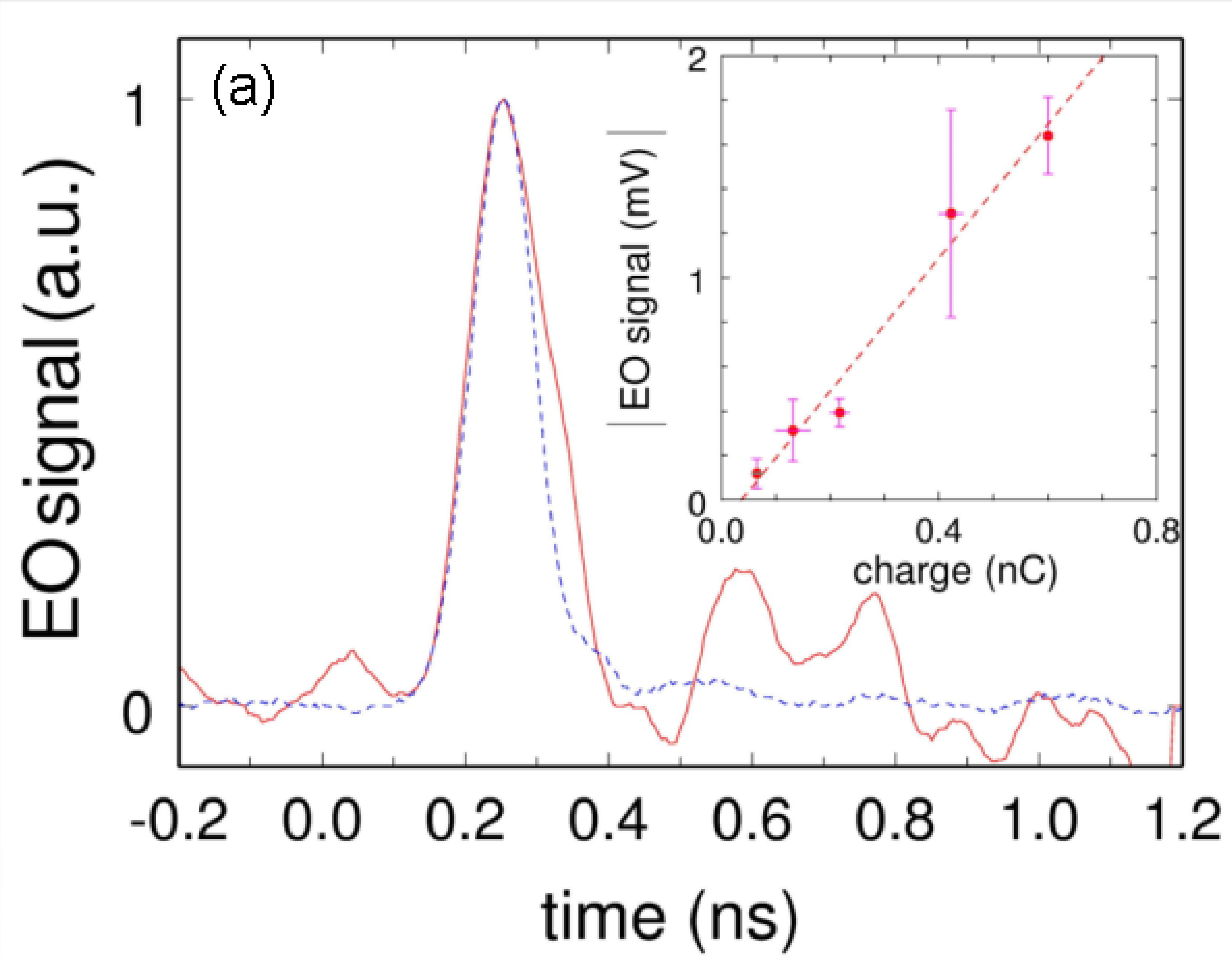
Fig. 1. The experimental setup for detecting a charged particle beam. The $LiNbO_3$ crystal (M) was located in vacuum several mm from the beam position which could be varied over several cm. The beam direction was perpendicular to the plane of the page. The positions of the polarization maintaining fibers, polarizer (P), lenses, $\frac{1}{4}$ wave plate ($\lambda/4$), analyzer (A), shield wall (S) and photodiode detector(PD) are schematically indicated.

Time response:

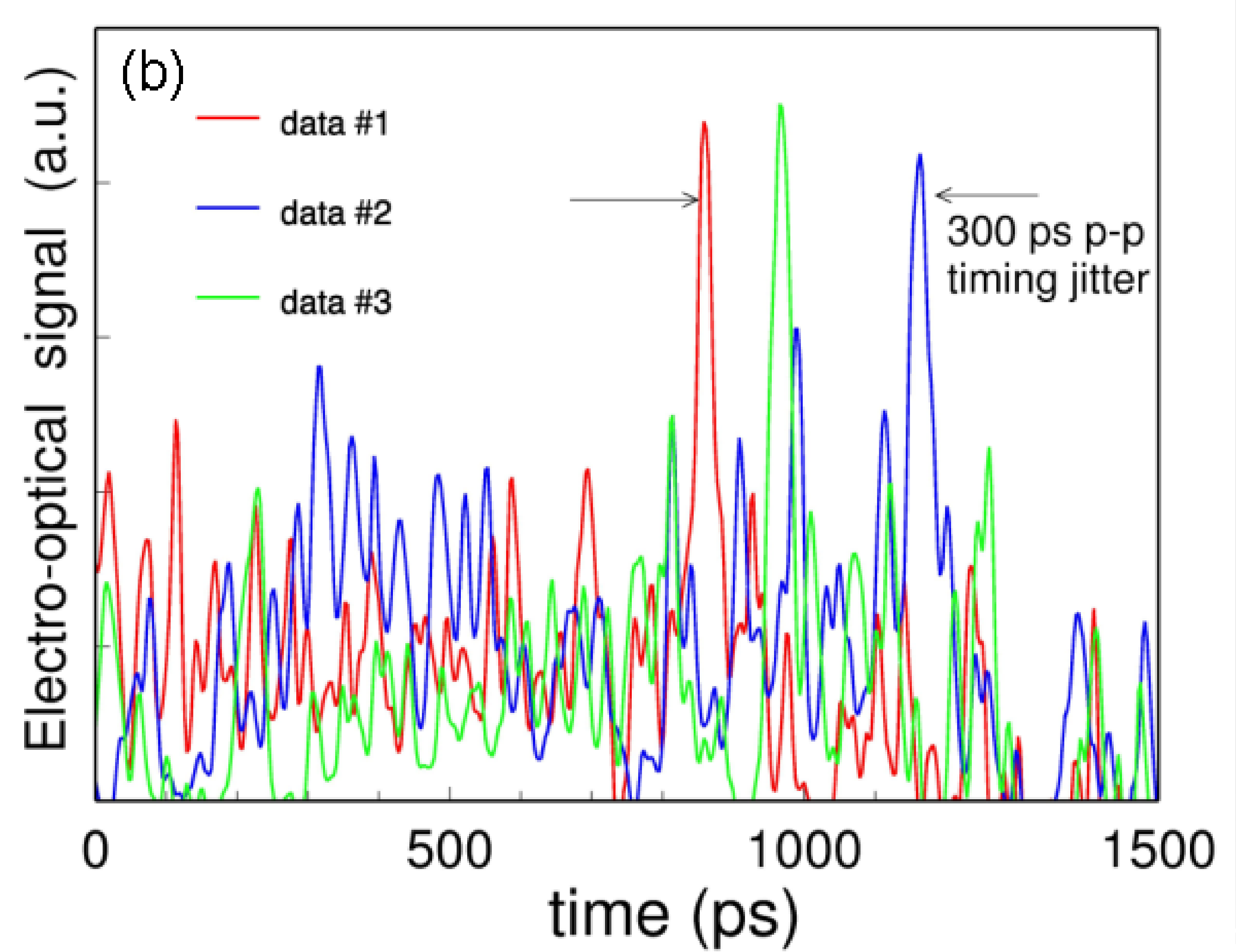
Single shot

CW laser

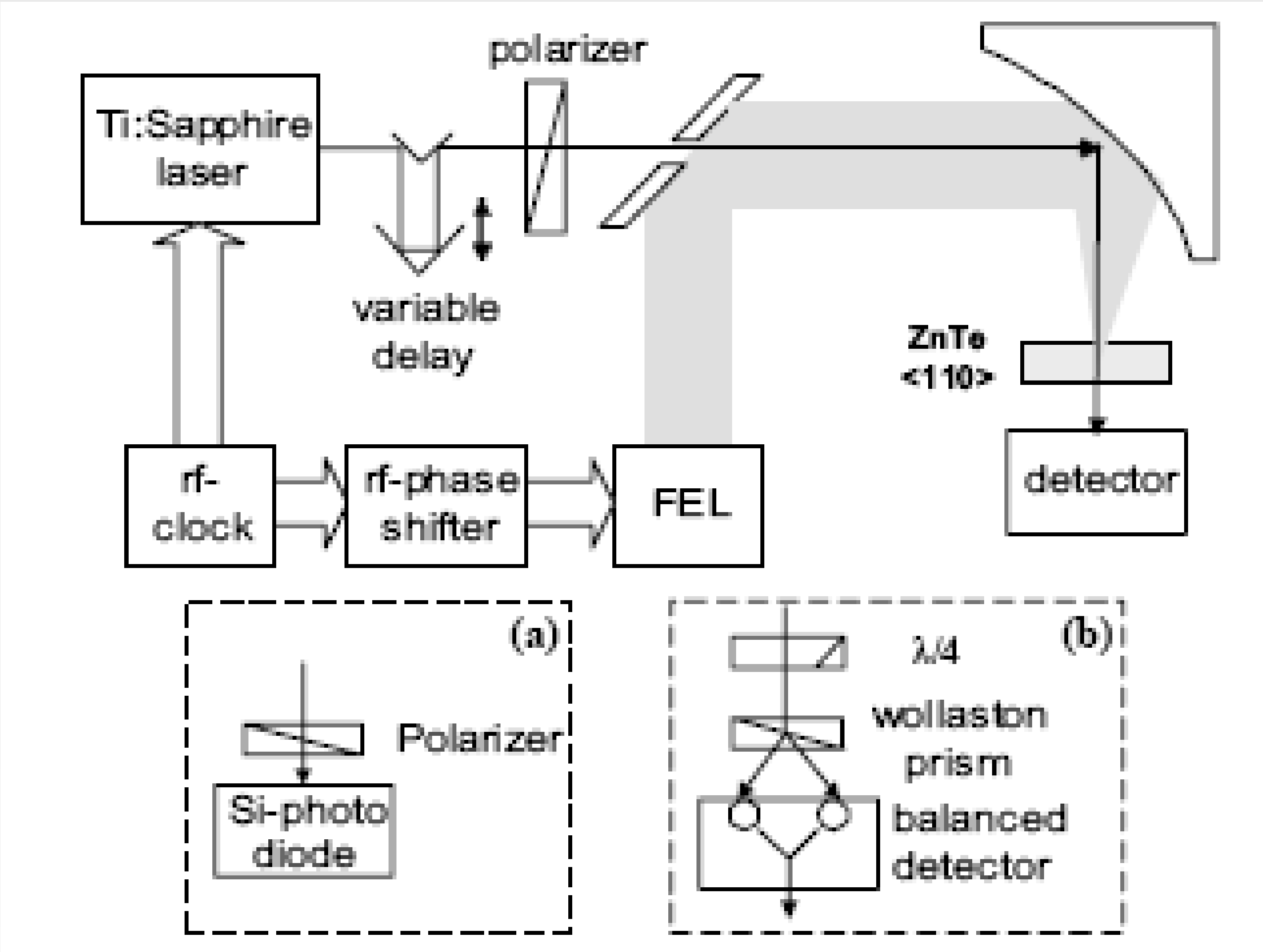
Courtesy *Nuclear Instruments and Methods in Physics Research A 452 (2000) 396–400*



Time resolution limited by 7 GHz scope



Free space Mach-Zehnder interferometric detection of the EO signals on a streak camera. All data are single-shot measurement results.

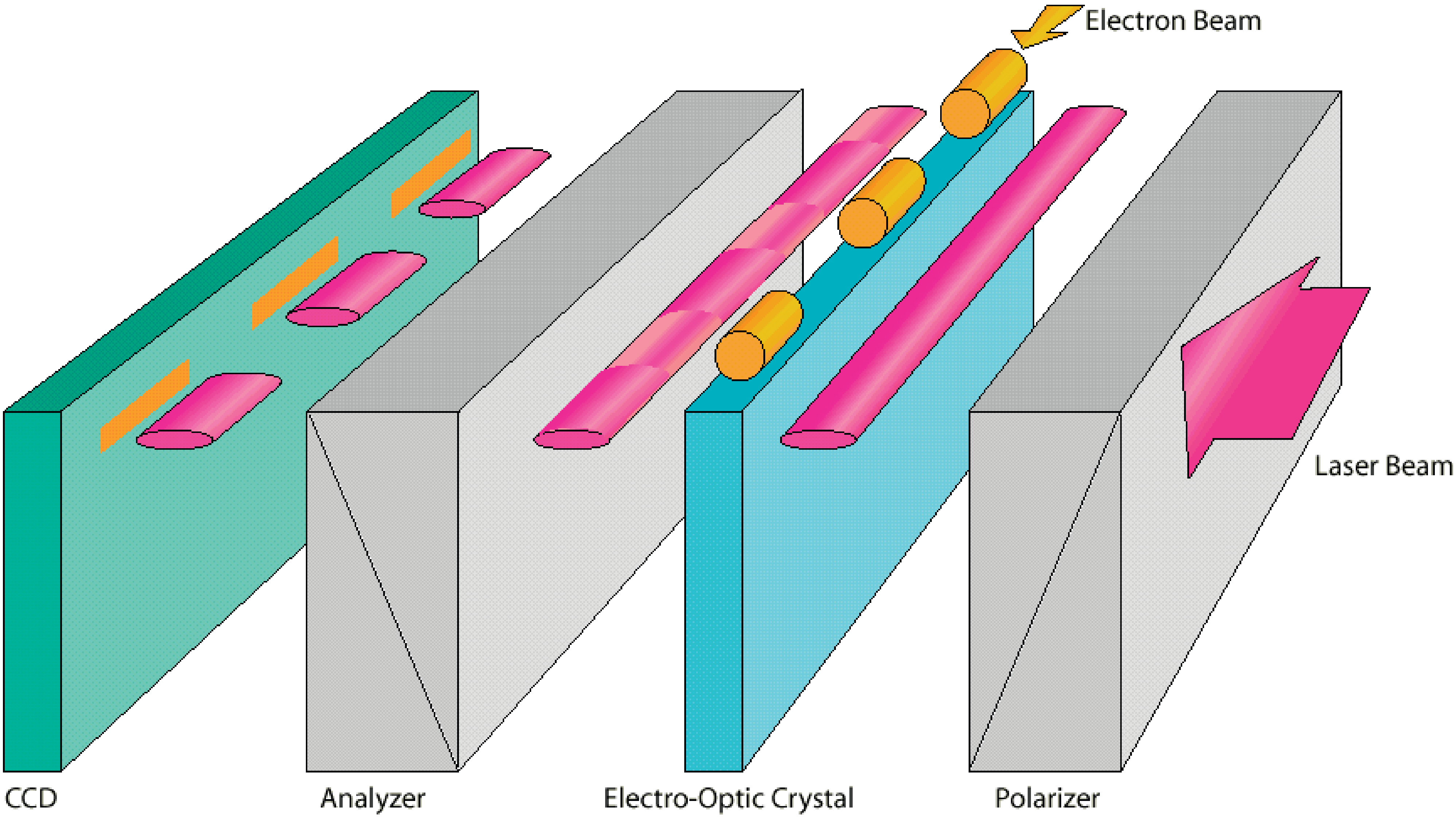


Time response;

Multishot

12 fs pulse laser

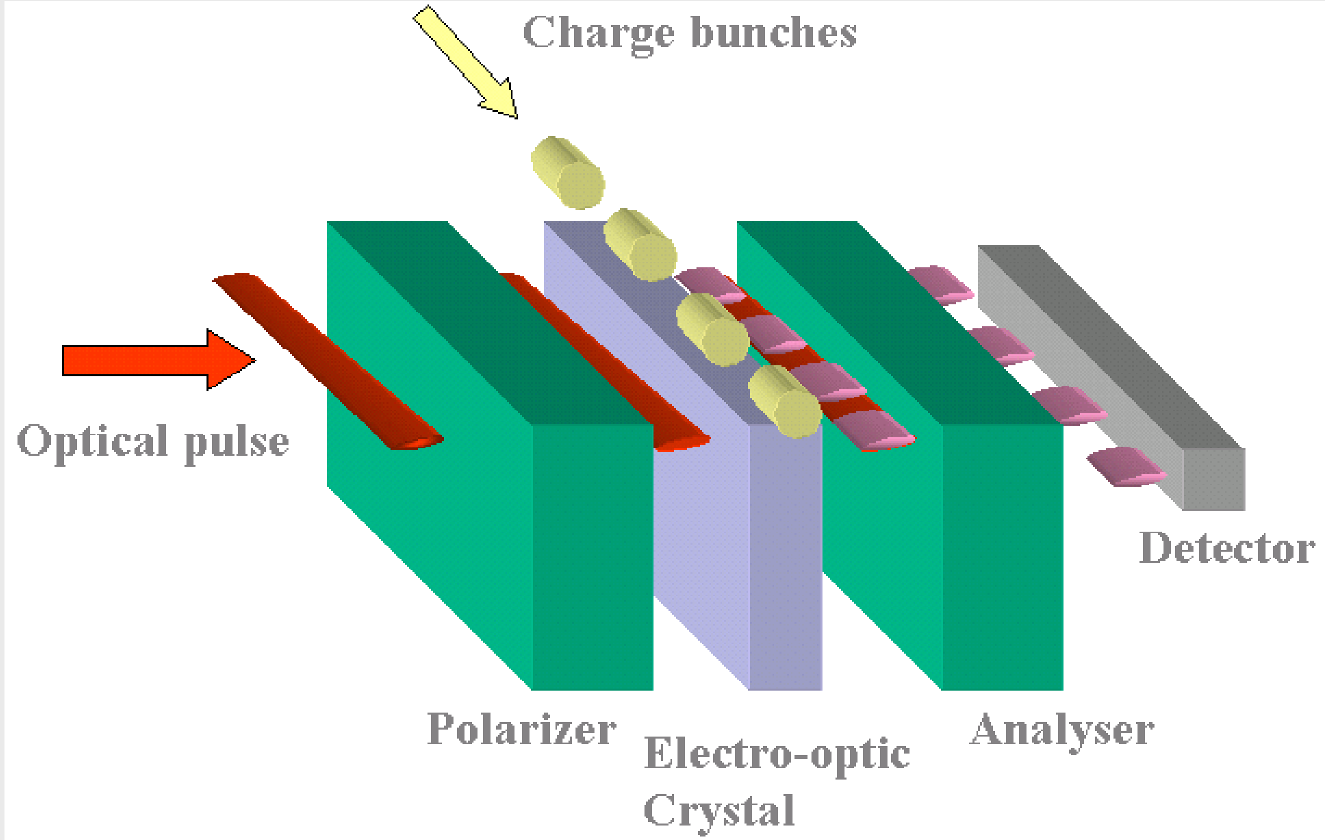
Courtesy: *Nuclear Instruments and Methods in Physics Research A 475 (2001) 504–506*


Electron Beam
Laser Beam
CCD
Analyzer
Electro-Optic Crystal
Polarizer

Spatial profile

Single shot

Short laser pulse



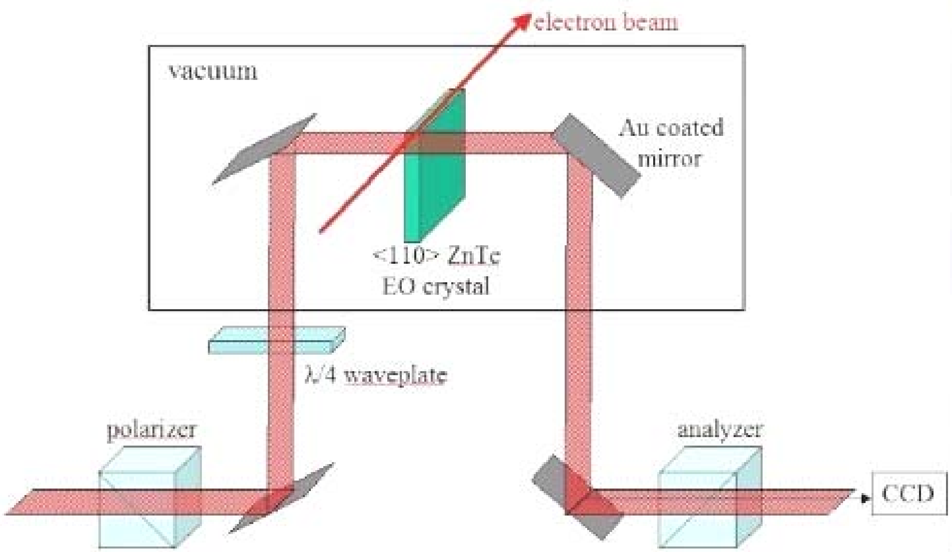


Fig. 1.3 Schematic of EO setup

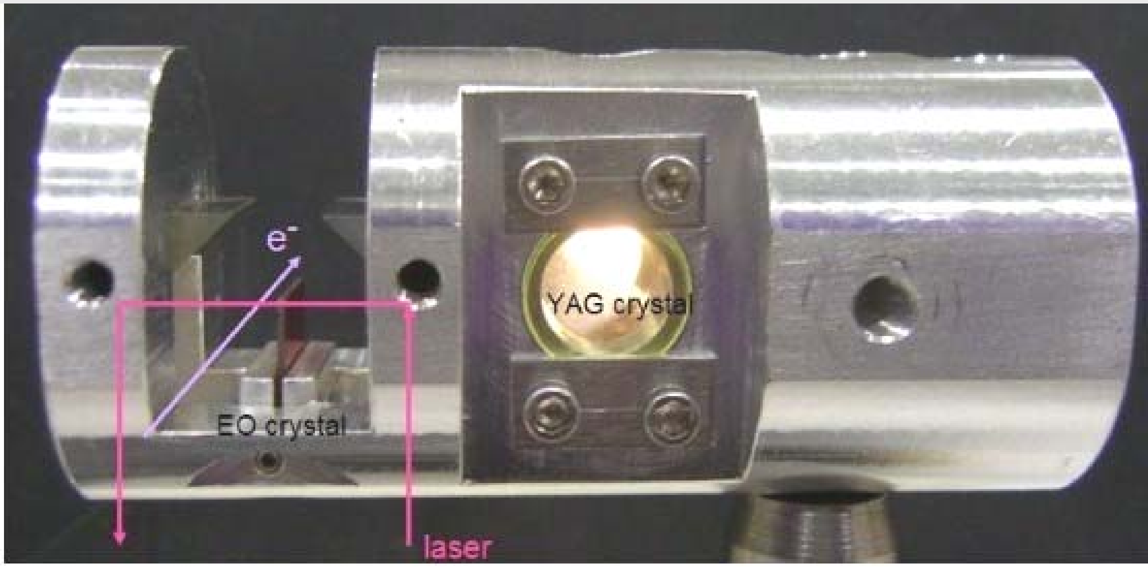
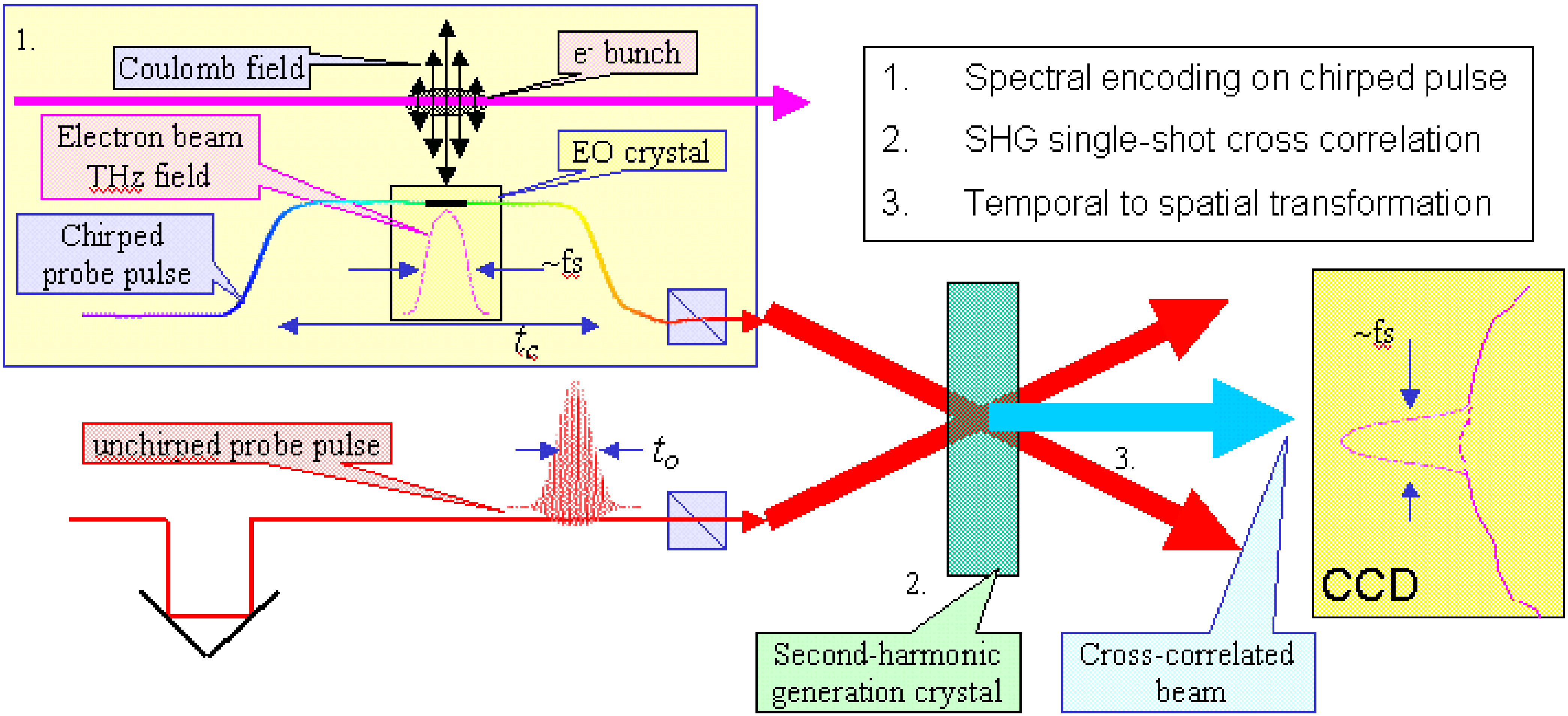


Fig. 1.4 EO-flash detection module

Cross correlation technique

Single shot

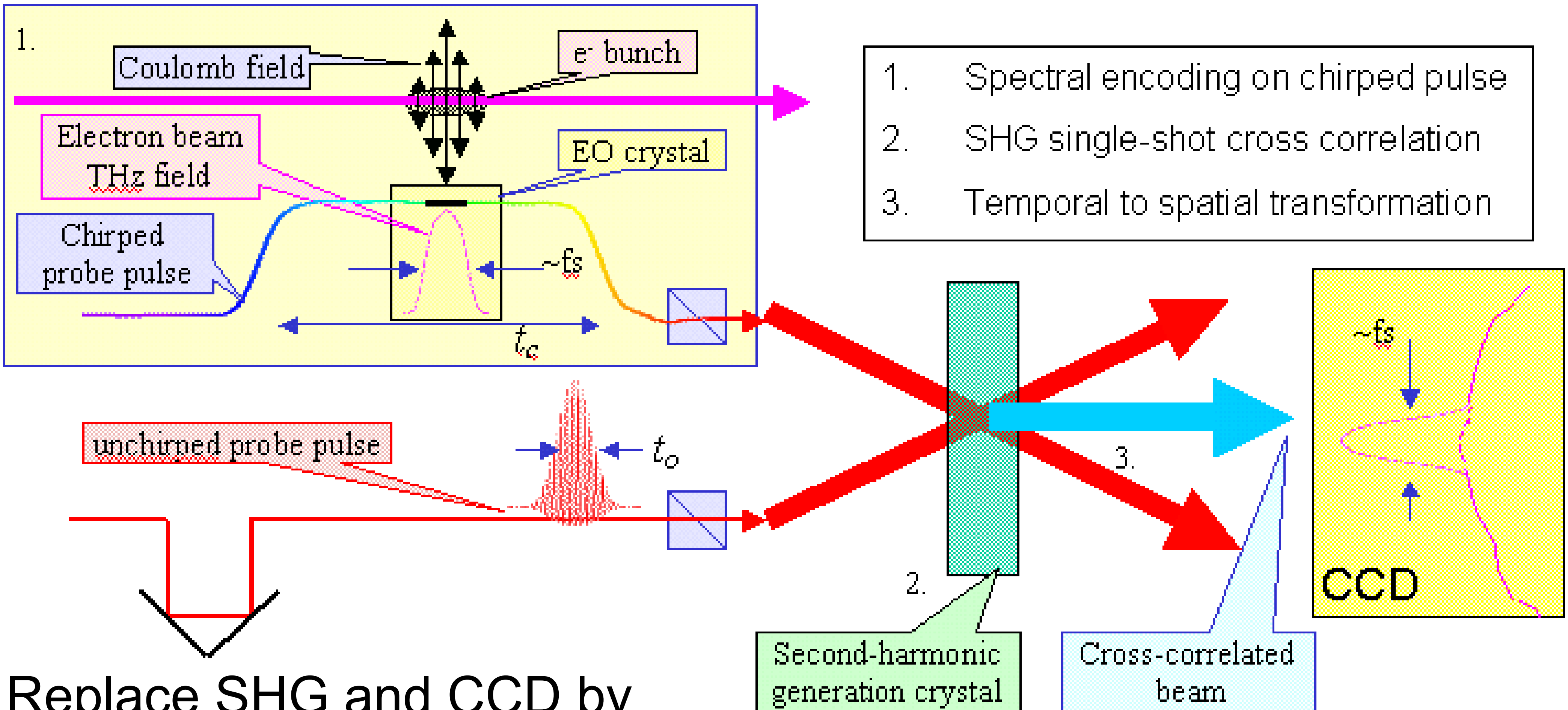
Short and chirped pulses



Spectral Response

Single shot

Chirped pulses

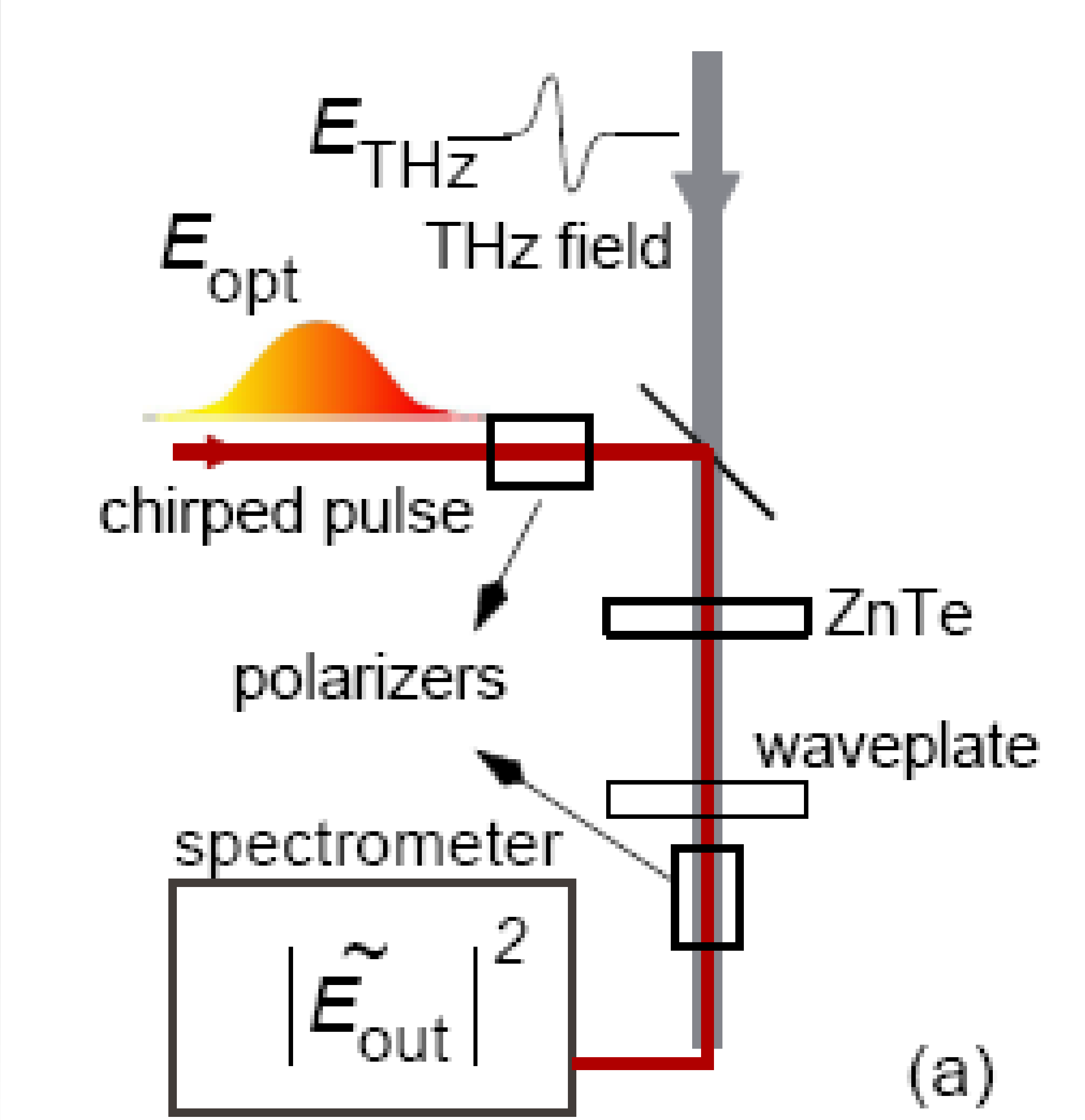


Replace SHG and CCD by spectrometer measure spectral distribution of transmitted beam through cross polarizer

Spectral content

Single shot

Chirped laser pulse



Triveni Rao, USPAS 2008, Annapolis

Limitations

- Crystal absorption in 10 THz regime:~ 100 fs
- Short distance between crystal and e beam