Shortening of a laser pulse with a self-modulated phase at the focus of a lens

Yuelin Li

Accelerator Systems Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

Robert Crowell

Chemistry Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

Received July 3, 2006; revised October 2, 2006; accepted October 5, 2006; posted October 10, 2006 (Doc. ID 72632); published December 13, 2006

We found that, at the focus of a chromatic lens, a laser pulse with a self-modulated phase can be shortened due to the radial dependence of the group delay imposed by the lens. Normally, this group delay stretches a short pulse into a long pulse by spreading the arrival time of the pulse at the focus. However, for a pulse with a self-modulated phase, it causes the fields with different phases to overlap, thus resulting in destructive interference that shortens the pulse. © 2006 Optical Society of America

OCIS codes: 320.5540, 260.0260, 190.5940.

Distortion of an ultrashort laser pulse due to lenses was first discussed by Bor in the framework of geometrical optics[1] and later by Bor and Horvath[2] and Kempe *et al.*[3] in the framework of wave optics. These distortions, mainly lengthening the pulse, are due to group velocity delay (GVDE) and group velocity dispersion (GVDI). The GVDE mismatch causes a delay between the phase velocity and the group velocity in the lens media, hence spreading the arrival time of the pulse at the focus. The GVDI effect causes the pulse to lengthen in the lens and can be very important for ultrashort pulses with large bandwidth. The effect was later verified by experiments.[4–6]

In this Letter, we describe a phenomenon that may occur in focusing optics, such as a lens or a focusing zone plate, caused by self-phase modulation (SPM) or a B-integral during the propagation of the laser pulse. We find that a relatively long pulse can be significantly shortened at the focus due to the GDVE effect, which lengthens a short pulse in the absence of the SPM. Similar to the lengthening effect, this shortening is most severe at shorter wavelengths where the refractive index has a stronger dependence on wavelength. In addition, the effect can also impact pulses with durations from the picsecond to nanosecond range, covering many applications involving intense ultraviolet (UV) beam manipulation, such as in modern photoinjectors[7] and inertial confinement fusion experiments.[8] The effect should also be evaluated for focusing beams in atomic and plasma physics applications and in microscopy and optical communications.

Our discussion is based on the elaboration of Fresnel diffraction formulation by Kempe *et al.*[3] For a flat-top circular beam traversing a lens, at the focus of the lens, Eq. (2) in Kempe *et al.*, which describes the optical field in the frequency domain, can be rewritten as

$$U(\omega) = 2\pi \int r\mathrm{d}rA(\omega)\Gamma(r,\omega), \tag{1}$$

$$\Gamma(r,\omega) = \exp\left(jk_l d + j\frac{k_a}{2f}r^2\right)\exp\left[-j(k_l - k_a)\frac{r^2}{2(n-1)f}\right]. \tag{2}$$

Here U is the frequency domain representation of the field at the focus; f is the focal length of the lens; r is the distance at the entrance from the optical axis; k_l and k_a are the wave vectors in the lens and air, respectively; and n is the refractive index in the lens at the nominal wavelength. The amplitude of the beam at the entrance of the lens is assumed to be homogeneous and has a circular shape with a field of $A(r,\omega)$ in the frequency domain. The lens transfer function is Γ, and its r-dependence derives from the thickness variation across the beam aperture. The time domain field can be obtained by an inverse Fourier transform of Eq. (1):

$$u(t) = 2\pi \int r\mathrm{d}r a * \gamma, \tag{3}$$

where the symbol * represents convolution, with $a = a(r,t) = F^{-1}A(r,\omega)$ and $\gamma = \gamma(r,t) = F^{-1}\Gamma(r,\omega)$. Following the elaboration in Kempe *et al.*[3] and ignoring the constant phase terms, we obtain for Eq. (2)

$$\Gamma(r,\omega) = \exp[jk(\beta T\Delta\omega + \chi T\Delta\omega^2)], \tag{4}$$

with

$$T(r) = d - \frac{r^2}{2(n-1)f}, \quad \beta = \frac{\mathrm{d}n}{\mathrm{d}\omega}, \quad \chi = \beta\frac{1}{\omega} + \frac{1}{2}\frac{\mathrm{d}^2 n}{\mathrm{d}\omega^2}. \tag{5}$$

Here d is the thickness of the lens at $r=0$.

For a Gaussian pulse $a = a_0\exp(-2\ln 2t^2/\tau^2)$ with no SPM, where t is the full width at half-maximum (FWHM), we obtain from Eqs. (3)–(5)

$$u(t) = 2\pi a_0 \int r\mathrm{d}r \frac{1}{(1+\Lambda^2)^{1/4}} \times \exp\left[-2\ln 2\frac{(t-k\beta T)^2}{\tau^2(1+\Lambda^2)}(1-j\Lambda)\right] \times \exp[j\tan^{-1}(-\Lambda)], \quad (6)$$

where $\Lambda = 4k\chi T/\tau^2$. In comparison with the input pulse, Eq. (6) shows pulse distortions discussed earlier that are due to both GVDE and GVDI effects.[1–3]

An interesting scenario not discussed before is when the pulse has an amplitude-dependent, thus time-dependent nonlinear phase, such as SPM. Assuming that the pulse has a small enough bandwidth that the second-order terms (Λ^2) in Eq. (5) can be neglected, we have

$$\gamma(r,t) = \delta(t - k\beta T). \quad (7)$$

Under this condition, the nonlinear Schrödinger equation[9] can be analytically solved for $a = a_0\exp(-2\ln 2t^2/\tau^2)$ to give the pulse after the lens as

$$a(r,t) = a_0\exp\left(-2\ln 2\frac{t^2}{\tau^2}\right)\exp\left[j\mu\exp\left(-4\ln 2\frac{t^2}{\tau^2}\right)\zeta\right]. \quad (8)$$

Here $\mu = kn_2a_0^2d$ is the maximum phase modulation, $\zeta = T/d$, if the SPM is generated in the lens, and $\zeta = L/d$ if the SPM is accumulated before arriving at the lens through a medium of an effective length of L. The pulse at the focus is then

$$u(t) = a_0 \int r\mathrm{d}r \exp\left[-2\ln 2\frac{(t-k\beta T)^2}{\tau^2}\right] \times \exp\left\{j\mu\exp\left[-4\ln 2\frac{(t-k\beta T)^2}{\tau^2}\right]\zeta\right\}. \quad (9)$$

We can see that, similar to Eq. (6), the integrand in Eq. (9) shows that the field at the focus is the superposition pulse slices with a shifted arrival time. However, the temporal dependence of the SPM now causes the superposition to be either constructive or destructive and, in general, shortens the pulse. This is depicted in Figs. 1(a) and 1(b), which show the real part of the integrand in Eq. (9) as a function of t and r. The drifting of the field pattern as function of r is clear, resulting in shortened pulses $u(t)$, in Figs. 1(c) and 1(d). The pulses' FWHMs are shortened to 0.21 and 0.2 ps, a factor of 5 reduction in comparison with the input pulse of 1 ps for the cases $\zeta = T/d$ and $\zeta = L/d = 1$, respectively. This shortening effect is in contrast to the lengthening effect discussed by previous authors.[1–3] Under the same conditions, a 50 fs pulse would be stretched to 0.5 ps without the SPM. Because of the similarity, in the following we limit our discussion for cases with $\zeta = 1$. In the calculation, we use a fused silica lens of $f = 150$ mm, $d = 5$ mm, and an aperture of $R = 12$ mm in radius (numerical aperture NA ≈ 1/6). The laser wavelength is 249 nm with

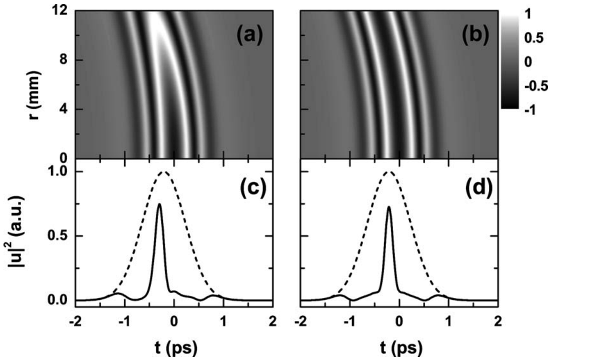

Fig. 1. Real part of the complex field arriving at the focus as a function of time and radius for (a) $\zeta = T/d$ and (b) $\zeta = 1$, and the corresponding intensity of the integrated field (solid curve) and the input pulse (dashed curve) as a function of time for the same cases [(c) and (d)]. The FWHM of the pulse is shortened from 1 ps to (c) 210 fs and (d) 200 fs, a reduction by a factor of 5. The calculation assumes an $f = 150$ mm lens with $R = 12$ mm and $d = 5$ mm. The pulse wavelength is 0.249 nm with $\mu = 15$ at a laser intensity of 5×10^{11} W/cm^2.

intensity of 5×10^{11} W/cm^2. The resulting μ is 15 rad. We use $n_2 = 2.38 \times 10^{16}$ W/cm^2.[10]

Because the pulse shortening is due to localized destructive superposition of the field, the phase slippage between the pulse slices traversing the lens should be big enough that destructive superposition dominates. However, the phase slippage should be small enough that the final pulse will not become a sum of quasi-random phasers, which can result in a thermal light.[11] Let the group delay between the lens center and edge be Δt: the above statement can be expressed as $\Delta t\ \mu/\tau \approx$ constant, or

$$\rho\mu \approx \text{constant}, \quad (10)$$

with

$$\rho = \frac{\beta R^2}{2(n-1)f\tau} = 2\mathrm{NA}\frac{\beta R}{\tau}. \quad (11)$$

The scaling in Eq. (10) is qualitatively demonstrated in Fig. 2, where the ratio of the pulse FWHM at the focus to that of the input pulse, calculated using Eq. (9), is plotted as a function of ρ and μ. Obviously, the maximum shortening parameter space centers around $\rho\mu = 2\pi - 3\pi$. Note that, although the plot in Fig. 2 is for an input pulse duration of 1 ps, we have verified that its characteristics are applicable for a large range of input pulse durations from a few femtoseconds to several nanoseconds if the GVDI is ignored.

This phenomenon can also be explained as the diffraction of a laser pulse with a distorted time-dependent wavefront. The pulse is shortened due to a time-dependent scattering. This is better understood by calculating the time and space dependence of the field, in which we again used Kempe's model[3] by including the SPM in the input pulse and the GVDI in the lens. Figure 3(a) shows the on-axis pulse envelope as a function of the distance from the focus Δz. Figures 3(b) and 3(c) give the intensity distribution

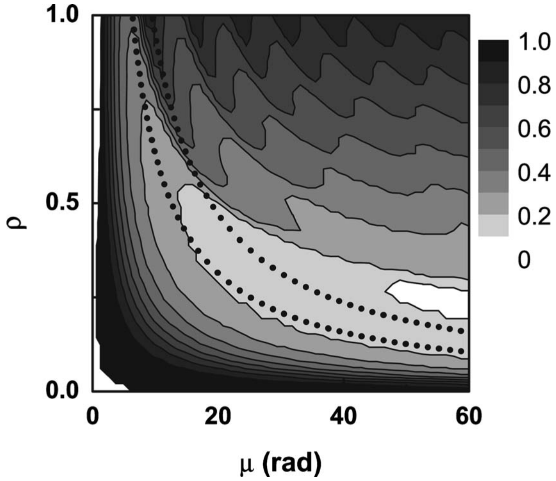

Fig. 2. Pulse duration ratio as a function of time-shift parameter ρ [defined in Eq. (11)] and the phase-shift parameter μ [defined in Eq. (8)] for a $\tau=1$ ps Gaussian pulse for $\zeta=1$. The two dotted curves are $\rho\mu=2\pi$ (lower) and $\rho\mu=3\pi$ (upper). Calculations for pulse duration ranging from a few femtoseconds to a few nanoseconds gives identical distributions when GVDI is ignored. The wavy structure is due to the generation of multiple peaks.

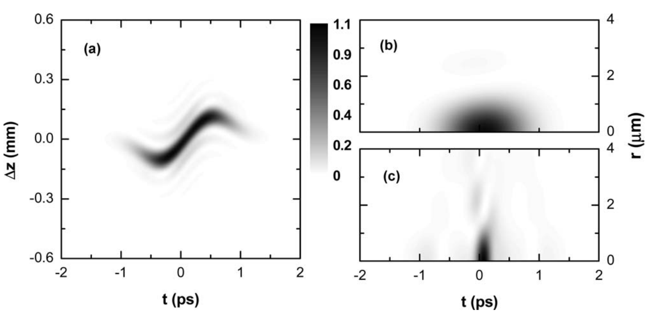

Fig. 3. On-axis laser pulse envelope as a function of the defocusing distance; the intensity as a function of time and radius for a pulse (b) without and (c) with the SPM effect. The calculation assumes the same conditions as for Fig. 1.

at the focus as a function of radius and time. Clearly, the shortened pulse duration is maintained beyond the Rayleigh range of 90 μm. The spatial fidelity of the pulse is also well maintained. It should be mentioned that the nonlinear phase can also be acquired through cross-phase modulation[9] when multiple laser beams overlap in time and space in the same medium and can be more severe than SPM.

As mentioned before, Eq. (9) is valid only when $\Lambda^2 \ll 1$ for both input pulse and the pulse with the modulated pulse. This limits the applicability in initial pulse duration, the medium length, and the laser pulse intensity. The laser intensity is, in addition, limited by the damage threshold of the medium, which is a few times $10^{12}\ \mathrm{W/cm^2}$ for fused silica.[12,13] For the examples in Figs. 1 and 3, the validity was checked by solving the nonlinear Schrödinger equation numerically and can be extended to input pulse duration down to 200 fs for the same lens and intensity.

We believe the pulse shortening due to this intensity-dependent pulse distortion needs to be carefully examined for applications involving manipulating intense UV beams, such as delivering high-quality UV pulses for a modern photoinjector[7] and focusing a multikilojoule UV laser beam into a hohlraum in inertial confinement fusion experiments.[8] In those applications, the modulated phase may accumulate during the laser transport and frequency conversion. The remedy is using achromatic optics when possible, which has been shown to be effective in mitigating the pulse lengthening effect.[1–3] It should also be noted this is a linear effect in the time domain because of a nonlinear phase in the laser pulse, whereas the more widely studied nonlinear pulse evolution effects are intertwined with spatial effects such as self-focusing, self-guiding, and the recently observed self-compression[14] and self-similar pulse collapse.[15]

The authors thank K. Harkay and K.-J. Kim for support. This work is supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, under contract W-31-109-ENG-38. Y. Li's e-mail address is ylli@aps.anl.gov.

References

1. Z. Bor, Opt. Lett. **14**, 119 (1989).
2. Z. Bor and Z. L. Horvath, Opt. Commun. **94**, 249 (1992).
3. M. Kempe, U. Stamm, B. Wilhelmi, and W. Rudolph, J. Opt. Soc. Am. B **9**, 1158 (1992).
4. Z. Bor, Z. Gogolak, and G. Szabo, Opt. Lett. **14**, 862 (1989).
5. R. Netz, T. Feurer, R. Wolleschensky, and R. Sauerbrey, Appl. Phys. B **70**, 833 (2000).
6. J. Jasapara and W. Rudolph, Opt. Lett. **24**, 777 (1999).
7. Y. Li and X. Chang, paper THPPH053 presented at the 2006 Free Electron Laser Conference, Berlin, August 27–September 1, 2006.
8. J. D. Lindl, Nucl. Fusion **39**, 825 (1999).
9. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, 1995).
10. A. J. Taylor, G. Rodriguez, and T. S. Clement, Opt. Lett. **21**, 1812 (1996).
11. J. W. Goodman, *Statistical Optics* (Wiley, 1985).
12. A.-C. Tien, S. Backus, H. Kapteyn, M. Murnane, and G. Mourou, Phys. Rev. Lett. **82**, 3883 (1999).
13. B. C. Stuart, M. D. Feit, A. M. Rubenchik, B. W. Shore, and M. D. Perry, Phys. Rev. Lett. **74**, 2248 (1995).
14. A. Couairon, M. Franco, A. Mysyrowicz, J. Biegert, and U. Keller, Opt. Lett. **30**, 2657 (2005).
15. K. D. Moll, A. L. Gaeta, and G. Fibich, Phys. Rev. Lett. **90**, 203902 (2003).