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Frequency-domain statistics of the chaotic optical field of a high-gain, self-amplified free-electron laser and its correlation to the time-domain statistics

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ABSTRACT Recently, the temporal evolution of the amplitude and phase of the chaotic output from a self-amplified free-electron laser was determined. In this letter, we characterize the frequency dependence of the amplitude and phase of the output radiation and discuss the relationship between the characteristics of the spiky intensity structures observed in both the time and the frequency domains. The distribution of the rms time–bandwidth products for an ensemble of output pulses is presented. Experimental results are compared with simulation and theory.

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Recently, there has been a renewed interest in applying the principles of statistical optics to characterize the chaotic radiation emitted by electron beams and other quantum radiation sources. Efforts in this direction include applying intensity interferometers [2] to the measurement of the spatial coherence [3] and pulse duration [4] of synchrotron X-ray radiation. Analysis of the statistical properties of such chaotic radiation in the frequency domain has been used to retrieve the charge profile of the electron beams that generate the synchrotron radiation [5]. There are also examples of characterizing the output of free-electron lasers (FELs) in the self-amplified spontaneous emission (SASE) mode, such as the pulse duration using spectral measurement in combination with the energy fluctuation [6].

The earlier measurements that we are referring to above, including the classical Hanbury-Brown–Twiss experiment [2], are time-integrated experiments. Work has been restricted to these types of measurements primarily due to the fact that natural thermal and chaotic light sources are intrinsically weak when filtered to reasonable coherence length/bandwidth. As a result, the correlation between time and frequency domains has thus far not been systematically studied experimentally.

The high intensity and high transverse coherence of a SASE FEL have made it possible to characterize the dynamic properties of the amplitude and phase in both the time and frequency domains. The analysis in the time domain reported in [7] characterized the temporal structure of the SASE FEL. It showed in great detail that the SASE output is a chaotic light source whose output is composed of a number of coherent modes each of which is represented by an intensity spike. The statistical properties of the amplitude and phase were shown to be well described by the central limit theorem (CLT) [8–11].

In this report, we extend our previous experimental analysis to the frequency domain and examine the correlation between the temporal and spectral structures. Supported by numerical simulations and theoretical analysis [8, 10], our work shows that the intensity spikes in the frequency domain can be identified as individual coherence regions and the spike width can be applied to determine the pulse duration of the chaotic radiation. Furthermore, we establish the correlation between the number of the spikes in the two domains. We have measured the rms time–bandwidth product (tbwp) for each of an ensemble of SASE pulses. Averaging the rms tbwp of the individual pulses over the ensemble determines the number of coherent modes. While our experiment was carried out at 530 nm, it is very relevant to SASE FEL radiation at X-ray wavelengths. The statistical properties we study are the foundation for characterizing the pulse using only spectral measurement, which will serve as a key single-shot measurement for future X-ray FELs based on the SASE principle.

The experimental setup has been described in previous publications. Briefly, the 530-nm output from the SASE FEL at the low-energy undulator test line (LEUTL) [12] at the advanced photon source is directed to a frequency-resolved optical gating (FROG) device [13], where the signal is $I_{\mathrm{FROG}}(\omega, \tau) \propto \left| \int_{-\infty}^{\infty} E(t)\, E(t-\tau)\, \exp(-\mathrm{i}\omega t)\, \mathrm{d}t \right|^2$, from which the amplitude and the phase of the input field E can be retrieved. Some key experimental parameters are the same as in [7]; only the electron bunch is shorter at 0.9 ps FWHM with 600 kA of peak current. The gain length is 0.77 m and the FEL is operating in the linear regime.

The behavior of the optical field is unpredictable from shot to shot. A representative example of both the raw and the reconstructed FROG traces, along with the field intensity and phase as

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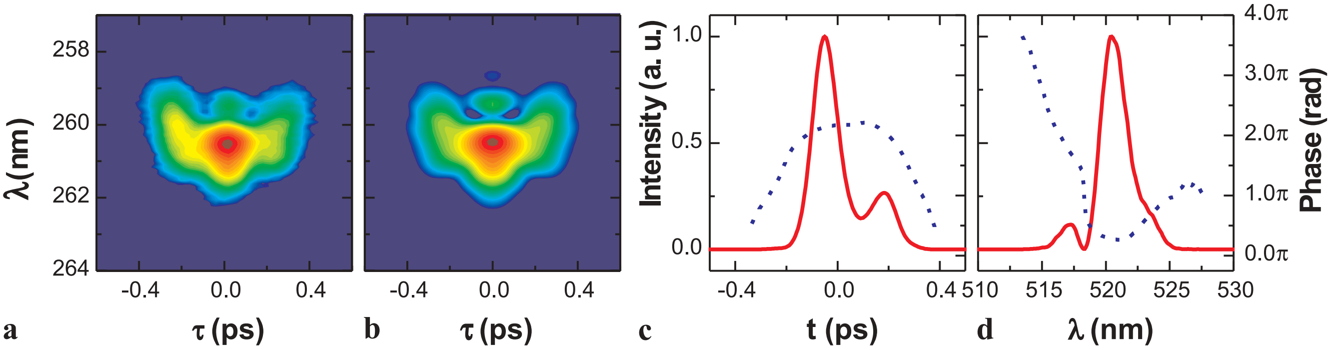

FIGURE 1 An example of a single-shot FROG measurement. (**a**) Raw data, (**b**) reconstructed image from FROG algorithm, (**c**) time-domain intensity and phase evolution, and (**d**) frequency-domain intensity and phase evolution. *Solid lines*: intensity; *dashed lines*: phase

a function of time and wavelength, are given in Fig. 1a–d. In this picture one sees that there are two intensity spikes in the time domain and two intensity spikes in the frequency domain. In the temporal domain, the envelope phase, intensity spike spacing, and spike width were studied before and good agreement was found with predictions from theoretical analysis of random noise [7]. We showed that the SASE FEL behaves as a stationary chaotic light source even though the FEL outputs are short pulses with durations of several hundred femtoseconds.

We can write the radiated electric field as $E(z,t) = A(z,t)\exp(\mathrm{i}k_\mathrm{r}z - \mathrm{i}\omega_\mathrm{r}t)$, where z represents the location along the undulator at which the SASE is observed and t represents the temporal position in the radiation pulse. In the case of an undulator with period $\lambda_\mathrm{u} = 2\pi/k_\mathrm{u}$ and magnetic field strength parameter K, the resonant frequency is $\omega_\mathrm{r} = k_\mathrm{r}c = 4\pi c\gamma^2/[\lambda_\mathrm{u}(1+K^2/2)]$. For a cold electron beam (zero energy spread), the SASE-radiated field before saturation (linear regime) can be approximated as the superposition of many electromagnetic wave packets emitted from randomly distributed, individual electrons [14–16]. In the frequency domain, the field amplitude can be written in the form

$$A(\omega,z) \cong A_0(z)\exp\left[-\frac{(\omega-\omega_\mathrm{r})^2}{4\sigma_\omega^2}\right] \times \sum_{j=i}^{N_\mathrm{e}} \exp\left(\mathrm{i}\omega t_j\right), \tag{1}$$

where N_e is the total number of electrons in the bunch, t_j is the random arrival time of the jth electron, $\bar{\sigma}_\omega = \omega_\mathrm{r}\sqrt{3\sqrt{3}\varrho/k_\mathrm{u}z}$ is the SASE gain bandwidth, and ϱ is the FEL parameter [10]. We have ignored the intrinsic chirp term [11, 18], which is not important for the analysis in this paper. In (1), the effect of the beam profile is not included. In our simulation, we work in the time domain and approximately include the beam-profile effect using a normal distribution for the arrival time and the dependence of gain on local density using an approach outlined in [17].

Similar to its counterpart in the time domain, (1) represents the field amplitude as a sum of random phasors, demonstrating that the statistical properties are determined by the central limit theorem (CLT). The experimental measurement of the spike properties using the FROG data verified this. In Fig. 2a and b we give the measured probability density distribution of the spectral intensity spike width and spacing. The distribution compares favorably with the analytical formula derived using

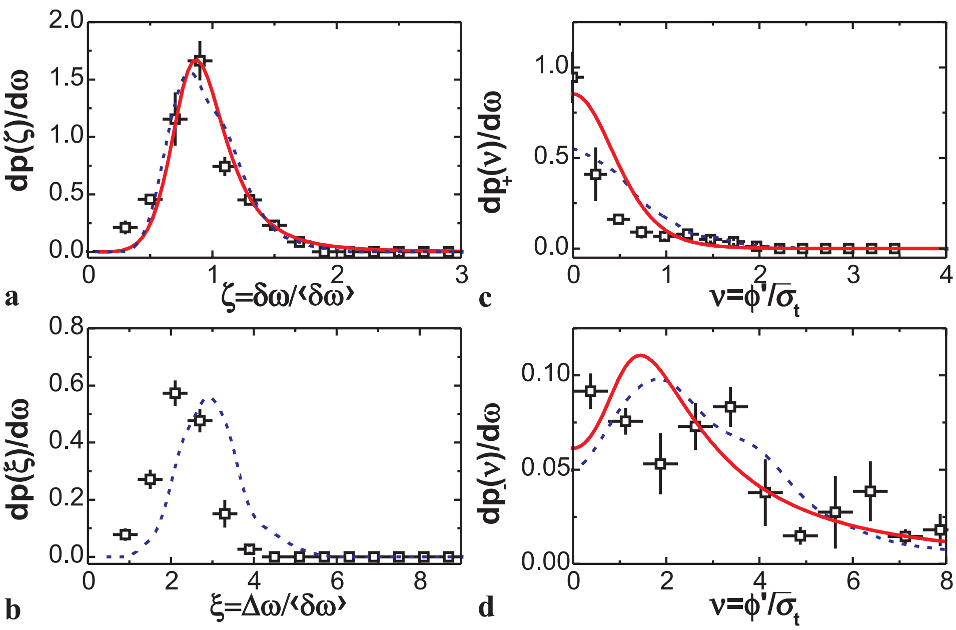

FIGURE 2 Measured probability distribution of (**a**) the spectral spike width $\delta\omega$, (**b**) the spike separation $\Delta\omega$, both normalized to the average spike rms width $\langle\delta\omega\rangle = 7.9$ mrad/fs; and probability distribution of the phase derivative $\varphi' = \mathrm{d}\varphi/\mathrm{d}\omega$ at the intensity maxima (**c**) and minima (**d**), normalized to the average rms pulse duration $\bar{\sigma}_\mathrm{t} = 83$ fs. *Symbols*: experimental data; *dashed lines*: simulation; *solid line*: theory using Rice's method [9]

Rice's method based on the CLT for a stochastic system [8–10], displayed in solid curves in Fig. 2. A good agreement with the simulation based on (1) is also observed. In the simulation 1000 simulated pulses having 400 electron macroparticles each were run using (1), with the beam-profile effect included as indicated in the text. The simulation is with a Gaussian electron bunch with an rms length of 420 fs. The FEL gain length is taken to be 0.77 m and the average rms bandwidth $\bar{\sigma}_\omega = 12$ mrad/fs. The electron-bunch length of 420 fs was chosen to yield the measured value of $2\bar{\sigma}_t\bar{\sigma}_\omega = 1.8$. The simulation generates data similar to the experimental data: $\bar{\sigma}_t = 73$ fs; $\langle\delta t\rangle = 48$ fs; $\langle\delta\,\omega\rangle = 8.3$ mrad/fs; $\langle\Delta t\rangle = 155$ fs; $\langle\Delta\omega\rangle = 26$ mrad/fs.

In Fig. 2c and d we show the probability density distributions of the phase derivative ($\mathrm{d}\varphi/\mathrm{d}\omega$), the dephasing rate of the field. The measurement was carried out at both the intensity maxima [Fig. 2c] and the minima [Fig. 2d]. The result is very similar to that obtained in the time domain [7]: the distribution of the phase derivative is very narrow at the intensity maxima but almost four times broader at the intensity minima. This clearly indicates that at the spectral intensity maxima, the phases are more correlated than at the spectral intensity minima, which are in the region where two different coherence regions connect. Our measurements demonstrate that the individual spikes are indeed coherence regions in the frequency domain.

To provide an approximate theoretical framework [10] in which to discuss the correlation between the time and frequency domains, let us assume the electron-bunch profile to be Gaussian but ignore the dependence of the gain on the local density. Also, although in the experiment the output pulse only contains a few intensity spikes, we shall employ the relations between quantities that hold when there are many spikes. Averaged over an ensemble of SASE pulses, the rms temporal width is denoted $\bar{\sigma}_t$ and the rms frequency spread is $\bar{\sigma}_\omega$. The coherence time is $T_{\mathrm{coh}} \cong \sqrt{\pi}/\bar{\sigma}_\omega$. The average spacing of the intensity spikes is $\langle\Delta t\rangle \cong \sqrt{2}T_{\mathrm{coh}}$ [8] and the rms spike width is $\delta\,t \cong T_{\mathrm{coh}}/\sqrt{2\pi}$ [8, 14–16]. In the frequency domain, the range of spectral coherence [11] is $\Omega_{\mathrm{coh}} \cong \sqrt{\pi}/\bar{\sigma}_t$. The average spacing of the spikes in the spectral intensity is $\langle\Delta\omega\rangle \cong \sqrt{2}\Omega_{\mathrm{coh}}$ and the rms spike width is $\langle\delta\,\omega\rangle \cong \Omega_{\mathrm{coh}}/\sqrt{2\pi}$.

		Measured	Calculated
Rms width	Time $\bar{\sigma}_t$ (fs)	83	
	Frequency $\bar{\sigma}_\omega$ (mrad/fs)	11	
Rms spike width	Time $\langle\delta\,t\rangle$ (fs)	52	$\langle\delta\,t\rangle = 1/\sqrt{2}\bar{\sigma}_\omega = 64$
	Frequency $\langle\delta\,\omega\rangle$ (mrad/fs)	7.9	$\langle\delta\,\omega\rangle = 1/\sqrt{2}\bar{\sigma}_t = 8.5$
Average spike spacing	Time $\langle\Delta t\rangle$ (fs)	208	$\langle\Delta t\rangle = \sqrt{2\pi}/\bar{\sigma}_\omega = 228$
	Frequency $\langle\Delta\omega\rangle$ (mrad/fs)	20	$\langle\Delta\omega\rangle = \sqrt{2\pi}/\bar{\sigma}_t = 30$
Coherence range	Time T_{coh} (fs)	$T_{\mathrm{coh}} = \sqrt{2\pi}\langle\delta\,t\rangle = 130$	$T_{\mathrm{coh}} = \sqrt{\pi}/\bar{\sigma}_\omega = 156$
	Frequency Ω_{coh} (mrad/fs)	$\Omega_{\mathrm{coh}} = \sqrt{2\pi}\langle\delta\,\omega\rangle = 19$	$\Omega_{\mathrm{coh}} = \sqrt{\pi}/\bar{\sigma}_t = 21$
Mode #	M	$M = 2\bar{\sigma}_\omega\bar{\sigma}_t = 1.8$	$M = 1/(\sigma_W/\langle W\rangle)^2 = 2.6$

TABLE 1 Correlation numbers between time and frequency domains

These approximate relations are compared with the experimental measurement in the time and frequency domains in Table 1, which gives both the experimentally measured value and the value calculated from its reciprocal counterpart ($t \leftrightarrow \omega$) – for the rms spike width, spike separation, and coherence range. The agreement is in general excellent. It is seen that the simple relations mentioned above (valid for a long pulse with many coherence regions) do provide a satisfactory first approximation to the experimental behavior. We note that for a short temporal pulse containing only a few spikes, measuring the average spike width $\langle\delta\omega\rangle$ yielded a better estimate of the pulse duration $\bar{\sigma}_t$ than did the measurement of the average spike separation $\langle\Delta\omega\rangle$.

Figure 3 depicts the correlation of the spike numbers in the time and frequency domains, where the size of the symbols on each point represents the normalized statistical weight. Due to the short bunch length of the electron beam, the highest spike number we have is three. Clearly, the correlation indicates a pattern centered on a one-to-one correspondence. As each of the intensity spikes represents a coherence region or mode, this measurement confirms that the number of coherence modes in the time domain statistically tends to equal the number of coherence modes in the frequency domain. We notice that this correlation has long been applied for determining temporal properties from spectral measurements for chaotic light sources, but has not previously been explicitly verified experimentally [4, 6].

Another important result from the theoretical analysis [10] is that the number of coherence modes M is found to be equivalent to the average rms time–bandwidth product (tbwp) of the radiation pulse, normalized by its minimum value of 1/2, i.e. $M = 2\bar{\sigma}_\omega\bar{\sigma}_t$. The fluctuation of output energy per pulse is determined by the mode number via $\sigma_W/\langle W\rangle = 1/\sqrt{M}$. In Fig. 4a we plot the probability density distribution of the normalized rms tbwp. The distribution averages at 1.8. In Fig. 4b, the distribution of the FEL pulse energy is plotted and compared with a gamma distribution with a mode number of 1.8. The mode number determined from the

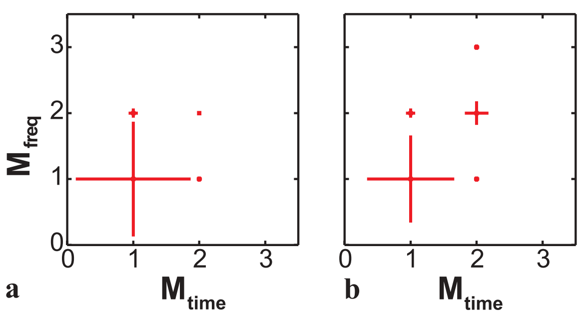

FIGURE 3 Correlation of the number of spikes in the time and frequency domains with two intensity ranges. (**a**) With spike peak intensity greater than 50% of the peak intensity of the largest spike and (**b**) with spike intensity greater than 5% of the peak intensity of the largest spike. The size of the *symbols* represents the normalized weights of the statistics on each data point

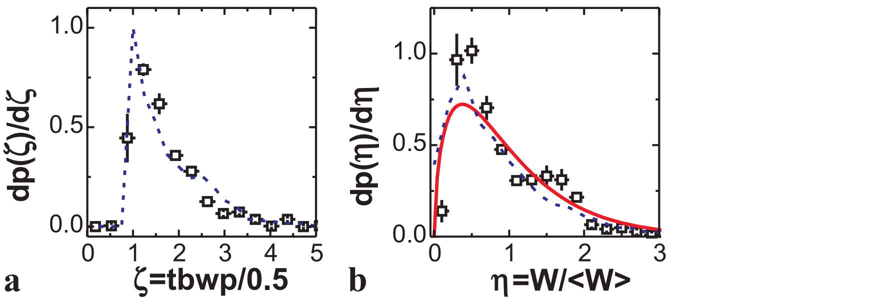

FIGURE 4 Probability distributions of (**a**) the normalized rms time–bandwidth product, tbwp/0.5 = $2\sigma_t\sigma_\omega$, where 0.5 is the transform-limited value, and (**b**) the pulse energy. *Symbols*: experiment; *dashed lines*: simulation; *solid line*: theory

measurement of the standard deviation of pulse energy is about 2.6. The relative energy was measured using the FROG trace, where only those pulses with retrievable traces could be used in the analysis. We believe that the deviation of the measured distribution from the expected form is due to this experimental limitation. Our data also show that the pulses with the highest energy have the smallest tbwp. We speculate that this has to do with the relative position of the spike in the bunch. Since the spike width is comparable to the bunch length, if a spike lands in the middle of the bunch it will dominate, and as a result the output will have one dominating, close-to-transform-limited spike. Otherwise there will be more spikes with lower intensities.

In summary, we stress that the frequency-domain behavior of a chaotic light source is statistically similar to the time-domain behavior. We explicitly verified that a high-resolution measurement of the average spectral spike width can be used to determine the temporal duration of the radiation. While our approximate theory provides a useful description of the correlation between the two domains, it remains as a challenge to future theoretical work to include the effect of the dependence of gain on the local density, and in particular to determine the temporal duration of the output radiation $\bar{\sigma}_t$ as a function of the electron-bunch length. To our knowledge, this is the first systematic study of the statistics of a stochastic system in the frequency domain and its correlation with the time-domain properties.

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